Math 3F03, Fall 2014 Midterm Exam 2

Name:

Student Number:

Instructions

• No calculators, notes, or books are permitted during this exam.
• This test consists of five questions and 25 points.
• All answers and work must be written in your exam booklet. **Make sure to put your name and student number on your exam booklet before handing it in**.
• For all questions you **must** show your work for full credit.
• You have 50 minutes.
• Good Luck!

1.) (5 pts) Compute the matrix exponential of $A = \begin{pmatrix} -6 & 4 \\ -9 & -6 \end{pmatrix}$

2.) (5 pts) Consider the IVP $x' = x + 2$, $x(0) = 2$.
(i)Write out the first three terms of the Picard iteration scheme for this IVP (you may write more terms if you wish).
(ii)Write an expression for $u_k(t)$, the k^{th} Picard iterate of this system. Your expression shoud *depend on t and k only*, and not on $u_{k-1}(t)$..
(iii)Use your answer in part (ii) to find a solution to this IVP.

3.) (5 pts) Consider the IVP $x' = |x|^{\frac{3}{2}+\sin a}$, $x(0) = 0$, $0 \leq a < 2\pi$. Give the interval in a for which
(i) the system has a *unique* solution
(ii) the system has a solution which is possibly *nonunique*.
Be sure to cite any theorems you use to reach your conclusions in parts (i) and (ii).

4.) (5 pts) Consider the system of equations
$x' = \cos x$
$y' = \sin y$.
(i) Find all equilibria of this system.
(ii) Find the linearized system around each equilibrium.
(iii) Do the linearized systems from (ii) accurately describe the local behavior of the full nonlinear system near the equilibrium points? Why or why not?

5.) (5 pts) Consider the system $X' = \begin{pmatrix} a & 0 & b \\ 0 & b & 0 \\ -b & 0 & a \end{pmatrix} X$. This system has the general solution
$X(t) = c_1 e^{at} \begin{pmatrix} \cos(bt) \\ 0 \\ -\sin(bt) \end{pmatrix} + c_2 e^{at} \begin{pmatrix} \sin(bt) \\ 0 \\ -\cos(bt) \end{pmatrix} + c_3 e^{bt} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. Indicate the regions of the ab-plane in which the phase portrait of this system is
(i) a spiral saddle
(ii) a spiral center
(iii) a source
(iv) a sink.