Math 3F03, Fall 2014
Assignment 5

This assignment covers material from the last few weeks of class. It will not be handed in nor graded, and is for practice only. However, doing this assignment will prepare you for the final exam questions on this material.
Page references refer to the course textbook, *Differential Equations, Dynamical Systems, and an Introduction to Chaos* by Hirsch, Smale, and Devaney, Third Edition.

1.) Page 212, exercise 9.

2.) Page 229, exercise 1.

3.) Page 232, exercise 16.

4.) Consider the IVP $y' = \frac{t-y}{2t+5y}$, $y(t_0) = y_0$. For which (t_0, y_0) can we expect this IVP to have a unique solution?

5.)Consider the planar system of ODEs

$$\begin{aligned} x' &= x(2 - x - y) \\ y' &= x - y \end{aligned}$$

(a) Find and classify all equilibria of this system.
(b) What is the index with respect to this system of the closed curve $x^2 + y^2 = 9$?

6.)Consider the planar system of ODEs

$$\begin{aligned} x' &= y \\ y' &= -x + y(9 - x^2 - 9y^2) \end{aligned}$$

Show that there must exist a closed orbit for this system on the region

$$S = \{(x, y) | 1 \leq (x^2 + y^2) \leq 9\}.$$

7.)Consider the system of ODEs $X' = AX$ where

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

If you are told that a periodic solution to this equation exists on some subset of $\mathbb{R}^2$, what can you say about the trace of A?

8.) Consider the planar system of ODEs

$$\begin{aligned} x' &= -x^3 + 2xy^2 \\ y' &= -y^3 \end{aligned}$$

This system has an equilibrium solution at $(0, 0)$. Prove that it is asymptotically stable.