

1. Evaluate the following integrals.

(a) $\int_1^2 x^5 \ln(x) dx$

(b) $\int \frac{x^2}{(x^2 + a^2)^{1/2}} dx$

(c) $\int \frac{1}{1 + e^x} dx$

(d) $\int (1 + \tan x)^2 \sec x dx$

2. Find the average value of the function $f(x) = \frac{1}{\sqrt{x}}$ on the interval $[1, 4]$. The Mean Value Theorem implies there exists c in $[1, 4]$ with $f(c) = f_{avg}$. What is the value c ?

3. Let $r > 0$. Does the improper integral converge or diverge? (Justify your answer.)

$$\int_{-\infty}^{\infty} \frac{r}{r^2 + x^2} dx.$$

4. Find the arclength of the curve $y = \ln(1 + x^2)$ for $0 \leq x \leq \frac{1}{2}$.

5. (a) The region under the curve $y = \cos^2(x)$ for $0 \leq x \leq \frac{\pi}{2}$ is rotated around the x -axis. Find the volume of the resulting solid.

(b) Find the volume of the solid if the region in part (a) is rotated about the y -axis.

6. (a) The curve $y = x^2$ for $0 \leq x \leq 1$ is rotated around the y -axis. Find the area of the resulting surface.

(b) Find the area of the surface if the curve in part (a) is rotated about the x -axis.

7. (a) Find all solutions of the differential equation $y' = -xe^y$.

(b) Solve the initial value problem $(1 + e^x)y' = e^x$, $y(0) = 0$.

(c) Solve the differential equation $xy' - 2y = x^2$, $x > 0$.

8. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is a solution to the initial-value problem $y' = x^2y - \frac{1}{2}y^2$, $y(0) = 1$.

9. A tank contains 100 L of pure water. Brine that contains 0.1kg of salt per liter flows into the tank at a rate of 10L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?

10. Do the following series converge or diverge? In case the series converges, determine its value.

(a) $\sum_{n=1}^{\infty} 3^{2+n} 2^{1-2n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 7n + 12}$

(c) $\sum_{n=0}^{\infty} \frac{n^2}{n^3 + 1}$

11. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is absolutely convergent, conditionally convergent or divergent.

12. Determine if the series $\sum_{n=3}^{\infty} \frac{e^{4n}}{(n-2)}$ converges or diverges.

13. Express the repeating decimal 4.14237237... as a fraction.

14. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ correct to four decimal points.

15. Derive the Taylor series of $f(x) = \sqrt{x-1}$ centered at $x = 2$. What is its radius of convergence?

16. Evaluate $\int \frac{e^x}{x} dx$ as an infinite series.