ArtsSci 1A06 **Review Problems for Test 1** **October 30, 2019**

1. Evaluate the following limit if it exists. Justify your answers! If the limit is infinite, work out whether it is $+\infty$ or $-\infty$. Otherwise, if the limit does not exist, write DNE and explain why not.

(a) $\lim_{x\to 4} \dfrac{2-\sqrt{x}}{4-x}$

(b) $\lim_{x\to 2} \left(\dfrac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x} \right)$

(c) $\lim_{x\to 2} \left(\dfrac{x^2-4}{x^3-8} \right)$

(d) $\lim_{x\to \infty} \dfrac{2x^2-5}{5x^3-7}$

(e) $\lim_{x\to 0} \dfrac{2x^2-5}{5x^3-7}$

(f) $\lim_{x\to 1} \tanh(x^2-1)$

(g) $\lim_{x\to \infty} \tan^{-1}(x-x^3)$

(h) $\lim_{x\to 0} \dfrac{\sin 2x}{x}$

2. Find y', and be sure to indicate what differentiation rules you use in each step.

(a) $y = \tan(1-x^2)$

(b) $y = \sqrt{4+\sin x \cos x}$

(c) $y = \ln(\ln(\ln x))$

(d) $y = \tan^{-1}(\sinh x)$

(e) $y = x^{e^x}$

(f) $\sec(xy) = x^2 - y$

3. Find all vertical and horizontal asymptotes for the graph of $y = \dfrac{2x-3}{5x+3}$.

4. Given $f(x)$ and $g(x)$ differentiable functions with $f(1) = 5,\ g(1) = 2,\ f'(1) = a$, and $g'(1) = b$, find the slope of the line tangent to $y = f(x)g(x^2) + f(x^2)g(x)$ at $(1, 20)$ in terms of the constants a, b.

5. Construct the linear approximation to $f(x) = \sqrt[3]{1+3x}$ at $a = 0$ and use it to approximate the value $\sqrt[3]{0.97}$.

6. Given a function $f(x)$ with $f'(2) = 13$, evaluate the limit $\lim_{x\to 2} \dfrac{f(x)-f(2)}{x^3-8}$.

7. A particle moves along a line with velocity $v(t) = t^3 - 9t$, where v is measured in meters per second. Find the displacement and distance traveled by the particle during the time interval $[1, 4]$.

8. A ladder 13 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.6 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 5 m from the wall?