**ArtsSci 1A06** **Review Problems for Test 2** **March 1, 2020**

1. (a) Determine the area of the region bounded by the curves $y = x - 1$, $y = \ln x$, and $x = e$.
(b) Determine the area of the region bounded by the curves $x = y^2 - 1$ and $x + 3y = 1$.

2. Find the average value $f_{avg}$ of the function $f(x) = \tan(x)$ on the interval $[0, \frac{\pi}{4}]$. The Mean Value Theorem implies there exists $c$ in $[0, \frac{\pi}{4}]$ with $f(c) = f_{avg}$. What is the value $c$?

3. Rocks are being lifted out of a quarry that is 40 meters deep using a thick cable whose mass is 1.2 kg/meter. How much work is required to lift a 500 kg load of rocks half way up from the bottom of the quarry?

4. (a) Find the volume of the solid obtained by rotating the region bounded by $y = e^{-2x}$ and $y = 1 + 4x - x^2$ for $0 \le x \le 2$ about the line $y = -2$.
(b) Find the volume of the solid if the region in part (a) is rotated about the $y$-axis.

5. A solid object has base given by the region in the plane enclosed by the parabola $y = 1 - x^2$ and the $x$-axis, and the cross-sections perpendicular to the $x$-axis are all squares. Find its volume.

6. Evaluate the following indefinite integrals.

(a) $\displaystyle\int 3\tan^{-1}\left(\frac{2}{x}\right)\,dx$

(b) $\displaystyle\int \frac{3}{2t^2 + 4t + 6}\,dt$

(c) $\displaystyle\int \tan^4(2u)\sec^4(2u)\,du$

(d) $\displaystyle\int t(t^2 - 4)^{3/2}\,dt$

(e) $\displaystyle\int x^7\cos(2x^4)\,dx$

(f) $\displaystyle\int \frac{t^2 + 8t}{(t+2)(t-1)(t-4)}\,dt$

(g) $\displaystyle\int \frac{5}{3 + \sqrt{x-2}}\,dx$

7. Evaluate the following definite integrals. Note that some might be improper, and in that case please also indicate whether the integral converges or diverges.

(a) $\displaystyle\int_2^3 \sqrt{9 - x^2}\,dx$

(b) $\displaystyle\int_1^\infty xe^{-2x}\,dx$

(c) $\displaystyle\int_1^2 \frac{2u}{\sqrt[3]{u^2 - 4}}\,du$

(d) $\displaystyle\int_0^2 \frac{1}{(2x-1)^3}\,du$

8. Use the Comparison Test to determine if the integral $\displaystyle\int_1^\infty \frac{x-1}{x^4 + 5x^2}\,dx$ converges or diverges. Justify your answer!

9. In this, you are only asked to set up the integrals, you do not need to evaluate them.
(a) Set up an integral for the length of $y = \sqrt{x+2}$ for $1 \leq x \leq 5$.
(b) Set up an integral for the length of $x = \cosh(x)$ for $0 \leq x \leq e$.
(c) Set up an integral for the surface area of the object obtained by rotating $y = \sec(x)$ for $0 \leq x \leq \frac{\pi}{4}$ about the $x$-axis.

10. (a) Find all solutions of the differential equation $y' = -xe^y$.
(b) Solve the initial value problem $(1 + e^x)y' = e^x,\ y(0) = 0$.