Mathematics 2R3 Test 2

Dr. Hart | Nov. 6, 2019

Name:______________________________

Student No.:______________________________

- The test is 50 minutes long.
- The test has 6 pages and 5 questions and is printed on BOTH sides of the paper.
- You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancies to the attention of the invigilator.
- Attempt all questions and write your answers in the space provided.
- Marks are indicated next to each question; the total number of marks is 25.
- You may use a McMaster standard Casio fx-991 MS or MS Plus calculator (no communication capability); no other aids are not permitted.
- Use pen to write your test. If you use a pencil, your test will not be accepted for regrading (if needed).

Good Luck!

Score

Question	1	2	3	4	5	Total
Points	5	5	5	5	5	25
Score						

continued ...

1. (5 marks) Put your answer in the space provided for each part.

   (a) The range of a linear transformation is a vector space. True or False.

   ____

   (b) The real vector space $\mathbf{R}^3$ and P_2, real polynomials of degree ≤ 2 are isomorphic. True or False.

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   (c) Suppose that $T : V \to W$ is a linear transformation, $\dim(V) = 6$, $\dim(W) = 3$ and $\text{nullity}(T) = 4$. What is the rank of T?

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   (d) If V is an inner product space and W is a finite-dimensional subspace not equal to V then the projection from V to W is one-to-one. True or False.

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   (e) The set $\{1, \sin(x), \sin(2x), \sin(3x, \sin(4x)\}$ is a linearly independent subset of $\mathrm{C}[0, 2\pi]$ with respect to the inner product $\langle f, g \rangle = \int_0^{2\pi} f g \, dx$. True or False.

   ____

continued . . .

2. (5 marks) In the inner product space of continuous functions on [-1,1] with the inner product given by

$$\langle f, g \rangle = \int_{-1}^{1} fg\,dx$$

find the projection of e^x onto the subspace generated by 1 and x.

continued ...

3. Let V be the inner product space of continuous functions on $[0, 2\pi]$ with inner product given by

$$\langle f, g \rangle = \int_0^{2\pi} fg \, dx.$$

(a) (3 marks) Compute the projection of x onto $\sin(2x)$ and $\cos(2x)$ in this inner product space.

(b) (2 marks) If the Fourier series for the function $f(x) = x^2$ is

$$\frac{4\pi}{3} - 4\pi\left(\sin(x) + \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) + \dots\right) + 4\left(\cos(x) + \frac{1}{4}\cos(2x) + \frac{1}{9}\cos(3x)\dots\right)$$

what is the Fourier series for $2 - x^2$?

continued . . .

4. Suppose that $v_1 = (2, 1)$ and $v_2 = (1, 1)$ and that $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation such that $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$.

   (a) (2 marks) Compute $T(0, 1)$.

   .

   (b) (3 marks) Write an expression for $T(x, y)$.

continued . . .

5. (a) (2 marks) Suppose that V and W are vector spaces and that $T : V \to W$. Define what it means to say that T is a linear transformation.

   (b) (3 marks) Suppose that U, V and W are vector spaces and $T_1 : U \to V$ and $T_2 : V \to W$ are linear transformations. Prove that $T_2 \circ T_1$ is a linear transformation.

THE END