Mathematics 2R3 Test 3

Dr. Hart Nov. 27, 2019

Name:______________________________

Student No.:______________________________

- The test is 50 minutes long.
- The test has 6 pages and 5 questions and is printed on BOTH sides of the paper.
- You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancies to the attention of the invigilator.
- Attempt all questions and write your answers in the space provided.
- Marks are indicated next to each question; the total number of marks is 25.
- You may use a McMaster standard Casio fx-991 MS or MS Plus calculator (no communication capability); no other aids are not permitted.
- Use pen to write your test. If you use a pencil, your test will not be accepted for regrading (if needed).

Good Luck!

Score

Question	1	2	3	4	5	Total
Points	5	5	5	5	5	25
Score						

continued ...

1. (5 marks) Put your answer in the space provided for each part.

   (a) If an $n \times n$ complex matrix A satisfies $Ax \cdot Ay = x \cdot y$ for all $x, y \in C^n$ then A is unitary. True or False.

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   (b) Compute the inverse of

$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \end{pmatrix}.$$

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   (c) If A is similar to B then B is similar to A. True or False.

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   (d) The eigenvalues of a normal matrix are real. True or False.

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   (e) The matrix

$$\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

   is unitary. True or False.

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continued . . .

2. (5 marks) Suppose that V is the subspace of differentiable functions on the real numbers generated by $\{1, x, e^{-x}, xe^{-x}\}$. Consider the linear operator D on V defined by $D(f) = f'$, the derivative of f. Display the matrix for D relative to the basis $B = \{1, x, e^{-x}, xe^{-x}\}$.

continued ...

3. Suppose that $T\colon P_2 \to P_2$ is the linear transformation given by

$$T(p(x)) = p(x+1)$$

where P_2 is the vector space of polynomials with complex coefficients of degree at most 2.

(a) (3 marks) Compute the determinant of T.

(b) (2 marks) Determine the eigenvalues and eigenvectors for T.

continued . . .

4. Suppose that A is the Hermitian matrix

$$A = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{i}{2} \\ 0 & -1 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

which has eigenvalues $1, 0$ and -1 and corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix}, \begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(a) (3 marks) Determine a matrix P which unitarily diagonalizes A.

(b) (2 marks) Compute A^{100}.

continued ...

5. (a) (2 marks) For a complex matrix A, define what it means for A to be normal.

   (b) (3 marks) Prove that if an n x n complex matrix A is unitarily diagonalizable and has real eigenvalues then A is Hermitian.

THE END