1. (5 points) Determine if the following statements are true or false.

(a) If $V$ is an $n$-dimensional vector space and $T : V \to V$ is a one-to-one linear operator then the range of $T$ is $V$

__________

(b) If $A$ and $B$ are similar then they have the same characteristic polynomial.

__________

(c) The real vector spaces of $3 \times 3$ real matrices and polynomials of degree at most 6 with real coefficients are isomorphic.

__________

(d) The eigenvalues of an Hermitian matrix are real.

__________

(e) If $T$ is an invertible linear transformation then its kernel is the zero subspace.

__________

continued . . .

2. (5 points) Place your answer on the line provided.

   (a) In an inner product space, if $u$ and $v$ are orthogonal then compute $\|2u - v\|$.

   ___________

   (b) Compute the length of $(-2, i)$ in $C^2$ with the usual Euclidean norm.

   ___________

   (c) A matrix cannot be similar to itself; true or false.

   ___________

   (d) Compute

$$\int_0^{2\pi} \cos(4x)\cos(3x)\,dx$$

   ___________

   (e) Name the conic section defined by

$$3x^2 + 4y^2 = 9.$$

   ___________

continued . . .

3. (5 points) Let $R^4$ have the standard Euclidean inner product. Use the Gram-Schmidt process to find an orthonormal basis for the subspace spanned by

$$u_1 = (1, 2, 2, 0), u_2 = (1, 3, 1, 1) \text{ and } u_3 = (1, 4, 0, 1)$$

continued . . .

4. (5 points)

(a) Suppose that $B = \{f_1, f_2, f_3\}$ is a basis for a subspace $V$ of real-valued functions defined on the real line where

$$f_1 = e^{-x},\ f_2 = xe^{-x} \text{ and } f_3 = x^2e^{-x}.$$

Let $D : V \to V$ be the linear operator differentiation with respect to $x$. Find the matrix for $D$ with respect to the basis $B$.

(b) Use the matrix from part (a) to compute $D(3e^{-x} + xe^{-x} - 2x^2e^{-x})$.

continued . . .

5. (5 points)Find a unitary matrix $P$ which unitarily diagonalizes $A$ and determine $P^{-1}AP$ where

$$A = \begin{pmatrix} 3 & -i \\ i & 3 \end{pmatrix}.$$

continued . . .

6. (a) (2 points) In the inner product space of continuous functions on $[-1, 1]$ with inner product given by

$$\langle f, g\rangle = \int_{-1}^{1} fg dx,$$

compute the inner product of 1 with $x^2$.

(b) (3 points) Suppose that $A$ is an $n \times n$ invertible matrix. Show that for $u, v \in R^n$,

$$(u^T A^T Av)^2 \leq (u^T A^T Au)(v^T A^T Av).$$

continued . . .

7. (5 points) Diagonalize the quadratic form

$$x^2 - 2y^2 - 2z^2 + 4yz$$

and say if it is positive definite, negative definite or indefinite.

continued ...

8. (5 points) Let A be a normal matrix. Prove that for all $x \in \mathbf{C}^n$ that $\|Ax\| = \|A^*x\|$.

THE END