# Mathematics 2R3 Practice Test 3

Dr. Hart

Last Name:______________________________ Initials:__________

Student No.:______________________________

- The test is 50 minutes long.
- The test has 6 pages and 5 questions and is printed on BOTH sides of the paper.
- You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancies to the attention of the invigilator.
- Attempt all questions and write your answers in the space provided.
- Marks are indicated next to each question; the total number of marks is 25.
- You may use a McMaster standard Casio fx-991 calculator (no communication capability); no other aids are not permitted.
- Use pen to write your test. If you use a pencil, your test will not be accepted for regrading (if needed).

**Good Luck!**

**Score**

| Question | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Points | 5 | 5 | 5 | 5 | 5 | 25 |
| Score | | | | | | |

continued . . .

1. (5 marks) Answer the following true or false and put your answer in the space provided.

   (a) If $A$ is an $n \times n$ real symmetric matrix then any two eigenvectors are orthogonal.

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   (b) If $A$ and $B$ are similar then they have the same characteristic polynomial.

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   (c) The eigenvalues of an Hermitian matrix are real.

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   (d) If $T$ is an invertible linear transformation then its kernel is the zero subspace.

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   (e) A matrix cannot be similar to itself.

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continued ...

2. (5 points) Suppose that $D : P^2 \to P^2$ is the differentiation operator $D(p) = p'$.

   (a) Find the matrix associated to $D$ relative to the basis $B = \{1, 1 + x, 1 + x + x^2\}$.

   (b) Use the matrix from part (a) to compute $D(3 + 2x + x^2)$.

continued ...

3. Suppose that $T : \mathbf{R}^3 \to \mathbf{R}^3$ is given by

$$T(x, y, z) = (x_1 - x_2, x_2 - x_3, x_3 - x_1).$$

(a) (2 points) Compute the determinant of $T$.

(b) (3 points) Determine the eigenvalues of $T$.

continued ...

4. (5 points) Find a unitary matrix $P$ which unitarily diagonalizes $A$ and determine $P^{-1}AP$ where

$$A = \begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}.$$

continued . . .

5. (5 points) Suppose that $A$ is a normal matrix. We know there is a unitary matrix $U$ and a diagonal matrix $D$ such that $A = U^*DU$.

   (a) Suppose that $\lambda$ is an eigenvalue for $A$ and $x$ is an eigenvector for $\lambda$. Show that $Ux$ is an eigenvector for $D$.

   (b) Assume as a fact that if $x$ and $y$ are eigenvectors for different eigenvalues of a diagonal matrix then $x \cdot y = 0$. Use this fact and part (a) to conclude that for a normal matrix, eigenvectors for distinct eigenvalues are orthogonal.

THE END