Assignment 2, Math 2S3

Due Feb. 4 in class

(1) (a) How many elements are in an n-dimensional vector space over the field with 2 elements, Z_2?
    (b) How many invertible 2 x 2 matrices are there over Z_2? How many invertible 3 x 3 matrices are there over Z_2? You can use the fact that for a matrix to be invertible, the columns must be linearly independent.
    (c) Write out an expression for the number of invertible $n \times n$ matrices over Z_2.

(2) Show that if U and W are finite-dimensional K-vector spaces, $\dim(U \times W) = \dim(U) + \dim(W)$.

(3) Suppose that U and W are finite dimensional subspaces of some K-vector space V. Compute the dimension of $U + W$. Hint: Begin with a basis for $U \cap W$.

(4) For $n \times n$ matrices over K, $M_n(K)$, a particularly interesting linear functional is the trace: if $A = (a_{ij})$ then

$$tr(A) = a_{11} + \ldots + a_{nn}$$

Show that for any two matrices $A, B \in M_n(K)$, $tr(AB) = tr(BA)$.