Assignment 1, Math 4L3

Due Sept. 19, in class

1. Show by induction on formulas that the number of right brackets in a formula is greater than or equal to the number of $\rightarrow$ symbols.

2. Read pages 24 – 26 of the Goldrei book to learn the definition of subformula and subformula tree and then construct the tree corresponding to
$$(\neg((p \wedge \neg q) \vee (q \rightarrow r)) \rightarrow s)$$
and list all of its subformulas.

3. In class, we described an algorithm for recognizing when a string of propositional symbols was a formula. I repeat the algorithm here: for a string $s$,

   (a) If $s$ is empty then it is not accepted.

   (b) If $s$ has a single symbol then $s$ is accepted iff $s$ is a propositional variable.

   (c) If $s$ has length greater then 1 and either doesn't begin with a $\neg$ or (, or $s$ does not have an allowable bracket count then $s$ is not accepted.

   (d) Otherwise, if $s$ has length greater than 1 and $s = \neg t$ then $s$ is accepted iff $t$ is accepted.

   (e) If $s$ begins with a ( then locate the first binary connective $\square$ with bracket count 1; if there is none, $s$ is not accepted.

   (f) If we have identified $\square$ then since $s$ has an allowable bracket count, $s$ has the form $(s_1 \square s_2)$ and in this case, $s$ is accepted iff $s_1$ and $s_2$ are accepted.

   Show that every formula is accepted by induction on formulas.