Assignment 2, Math 4L3
Due Oct. 5, in class

1. Write out the proof indicated in class that every formula is equivalent to one in conjunctive normal form.

2. For your soul, as Goldrei says, write out the formal derivations of the following:

   (a) $\neg\neg\varphi \vdash \varphi$ for any formula φ

   (b) $\vdash (\varphi \to \varphi)$ for any formula φ

3. In class we proved the deduction theorem. In fact, we gave an algorithm for converting a proof of ψ from Γ and φ into a proof of $(\varphi \to \psi)$ from Γ. Give an upper bound on the length of the second proof in terms of the length of the first.

4. The completeness theorem tells us that if $\Gamma \models \varphi$ then there is a finite $\Gamma_0 \subseteq \Gamma$ such that $\Gamma_0 \models \varphi$. It is a good exercise to try to think how to prove this without the completeness theorem; what is the problem? Without the completeness theorem, show that the following are equivalent:

   (a) For every Γ, if every finite subset of Γ is satisfiable then Γ is satisfiable.

   (b) For every Γ, if $\Gamma \models \varphi$ then there is a finite $\Gamma_0 \subseteq \Gamma$ such that $\Gamma_0 \models \varphi$.