Assignment 4, Math 4L3
Due Nov. 21, in class

1. For the following structures in the indicated languages, find a sentence which is true in one but not the other.

   (a) $(\mathbf{N}, <)$ and $(\mathbf{Z}, <)$ in a language with a single binary relation symbol.

   (b) (Read the discussion on page 195 of Goldrei) As Boolean algebras, the full power set $\mathcal{P}(N)$ for the natural numbers N and the following collection of subsets of N, B, which I now describe: for a prime p, let
   $$U_p = \{n \in N : p \text{ divides } n\}$$
   and let B be all finite unions, intersections and complements of the sets U_p as p ranges over all primes. The operations on $\mathcal{P}(\mathcal{N})$ and B are given by union, intersection and complement. Hint: consider the order given by $b \leq a$ if $a \cap b = b$ i.e. $b \subseteq a$. We say that an element a is an atom if whenever $b \leq a$ then $b = a$ or $b = 0$. Ask yourself if every element of these algebras has an atom which is less than or equal to it.

2. Suppose that $\mathcal{M}$ and $\mathcal{N}$ are two L-structures and $f : \mathcal{M} \to \mathcal{N}$.

   (a) Show that if f is an embedding then for all quantifier-free formulas φ and all assignments of variables i in M,
   $$\mathcal{M} \models_i \varphi \text{ iff } \mathcal{N} \models_{f(i)} \varphi.$$

   (b) Show that if f is an isomorphism then it is an elementary map.

3. Show that $(\mathbf{R}, <) \not\cong (\mathbf{R} \setminus \{0\}, <)$.

4. Use the fact that any two countable dense linear orders without endpoints are isomorphic to show that any two countably infinite dense linear orders with both a right and left end point are also isomorphic.