

Math 4L3, Midterm Test

Bradd Hart, Oct. 28, 2016

Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

1. (8 marks)

- (a) If the formula $(\varphi \rightarrow \psi)$ is a contradiction (always false), what can you say about φ and ψ ?
- (b) Show that if $\Gamma \vdash (\varphi \rightarrow \psi)$ then $\Gamma, \varphi \vdash \psi$.
- (c) Show that if $\Gamma \models \varphi$ and $\Delta, \varphi \models \psi$ then $\Gamma, \Delta \models \psi$.

2. (8 marks)

- (a) State the compactness theorem for propositional logic.
- (b) If the set of formulas

$$\{\neg\varphi_1, \neg\varphi_2, \dots\}$$

is inconsistent show that there is some N such that $\bigvee_{i=1}^N \varphi_i$ is a tautology.

3. (6 marks) Can you obtain all possible truth tables in any number of propositional variables using only formulas constructed with the connectives \wedge and \vee ?

4. (8 marks)

- (a) Define the notion of a term for a first order language L .
- (b) Why do terms have only finitely many free variables?