Assignment 4, Math 701
Due Dec. 4, in class

1. We consider the ideal structure for the ring $Z[x]$.
   (a) (Optional) Show that every ideal I in $Z[x]$ has the following form: there is a number k, polynomials $p_i \in Z[x]$ for $i \leq k$ and numbers $r_0, \ldots, r_k$ such that:
      i. $r_k | r_{k-1} \ldots | r_0$,
      ii. $\deg(p_i) \leq i$ and the coefficient of x^i in p_i is r_i for $i \leq k$, and
      $$I = p_0 Z[x] + p_1 Z[x] + \ldots + p_k Z[x].$$
   (b) Show that $Z[x]$ is not a principal ideal domain.
   (c) Give an example of a finitely generated module over $Z[x]$ which cannot be written as a sum of cyclic submodules.
2. Show that $M_n(D)$ is a simple ring for a division ring D i.e. it has no proper, non-zero ideals.
3. Show that Q_8 has no degree 2 faithful representations over R. (See example 7 page 845 for Q_8 and question 10, 18.2 for a hint.)
4. Is it true that for a division ring D, that the identity map is an isomorphism between $M_n(D)^{op}$ and $M_n(D^{op})$?