Lecture 6: Related rate problems

In certain practical problems, x and y are related by an equation and can be regarded as functions of a third variable t, which often represents time. Then implicit differentiation can be used to relate $\frac{dx}{dt}$ and $\frac{dy}{dt}$. These problems are related as *related rate problem.*

Example 1. A tiny spherical balloon is inserted into a clogged artery and is inflated at a rate of $0.002\pi\ mm^3$/min. At what rate is the radius of the balloon growing when the radius is $r = 0.005$ mm?

Solution: Recall that the volume of a ball is $V = \frac{4}{3}\pi r^3$. Hence,

$$\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}.$$

We know that $\frac{dV}{dt} = 0.002\pi$ when $r = 0.005$. Therefore,

$$0.002\pi = \frac{4}{3}\pi(3(0.005)^2)\frac{dr}{dt}.$$

Thus, $\frac{dr}{dt} = 20$mm/min.

Example 2. When the price of a certain commodity is p dollars per unit, consumers demand x hundred units of the commodity, where

$$75x^2 + 17p^2 = 5300.$$

At what rate is the demand x changing with respect to time when the price is \$7 and decreasing at a rate of 75 cents per month?

Solution: The question tells us that $\frac{dp}{dt} = -0.75$ when $p = 7$. Differentiating the given equation.

$$150x\frac{dx}{dt} + 34p\frac{dp}{dt} = 0.$$

When $p = 7$, $75x^2 + 17(7)^2 = 5300$ gives $x^2 = 59.56$. Putting in the information,

$$150(\sqrt{59.56})\frac{dx}{dt} + 34(7)(-0.75) = 0.$$

$\frac{dx}{dt} = 0.15$

Exercises:

1. A ladder of 5 metres long is leaning against a wall and is sliding down. Suppose that the top of the ladder is sliding down constantly at a rate of 0.9 m/s. How fast is the foot of the ladder moving away from the wall when the top is 3 m above the ground.

2. An ice block used in a cooler at campsite is modelled as a cube of side length s. The block currently has volume 125 000 cm^3 and is melting at a rate of 1000 cm^3 per hour.

(a) At what rate the side length is changing currently.

(b) What is the rate of change of the total surface area?

(Volume of a cube = s^3 and surface area = $6s^2$)

3. A circular oil is spreading through in water in such a way that its radius is increasing at a rate of 7 m/h. How fast is the area increasing when the radius is 50m.

4. A 1.8 metre tall man is walking at a rate of 1.5 m/s away from the base of a street light that is 3.6 m above the ground. At what rate is the length of his shadow is changing when he is 6 m away from the base of the light?