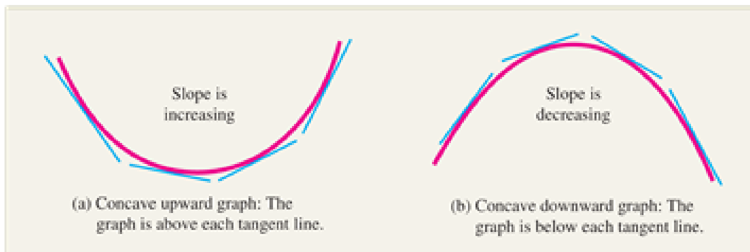
Lecture 7: Curve sketching: Second derivatives

We now describe the behavior of a function from the information about **second derivatives.**

$f(x)$ is said to be *concave upward (convex)* on the interval $[a, b]$ if $f'(x)$ is increasing on $[a, b]$.
$f(x)$ is said to be *concave downward (concave)* on the interval $[a, b]$ if $f'(x)$ is decreasing on $[a, b]$



Theorem 1.
(i) $f(x)$ is concave upward (convex) on the interval $[a, b]$ if and only if $f''(x) > 0$ on $[a, b]$.
(ii) $f(x)$ is concave downward (concave) on the interval $[a, b]$ if and only if $f''(x) < 0$ on $[a, b]$.

$(c, f(c))$ is called a *point of inflection* of $f(x)$ if $f(x)$ changes its concavity at c.

Theorem 2. *If $(c, f(c))$ is called a point of inflection of $f(x)$, then $f''(c) = 0$.*

Theorem 3 (Second derivative test). *Suppose that $f(c) = 0$. Then*
(i)If $f"(c) > 0$, then f has a relative minimum at $x = c$.
(ii) If $f"(c) < 0$, then f has a relative maximum at $x = c$.
(iii) However, if $f"(c) = 0$, the test is inconclusive and f may have a relative maximum, a relative minimum, or no relative extremum at all at $x = c$.

Exercises.
1. For the following functions, find the intervals for which it is concave up or concave down. Find also the points of inflection.
(i) $f(x) = x^3 + 3x^2 + 1$, (ii) $f(x) = x^5 - 5x^4 + 10$,
(iii) $f(x) = \frac{x}{x^2+3}$, (iv) $f(x) = \frac{x+1}{(x-1)^2}$.
(v) $f(x) = \frac{x^2-9}{x^2+1}$.