

1K03E, Test 3

Date: 12 June 2013,

Duration: 90 Minutes

Name :

Student ID :

Instruction: Please put down your multiple choice answers in the space below.

1	2	3	4	5	6	7	8

1. Let $y^2 = 2x^3$. Find $\frac{dy}{dx}$.

A. $\frac{3x^2}{2y^2}$

B. $\frac{3x^2}{2y}$

C. $\frac{3x^2}{y}$

D. $\frac{3x^3}{y}$

E. $\frac{3x^3}{2y^2}$

2. Let $y^2 = x^3 - 4x + 1$. Find the slope of the tangent line at the point $(-2, 1)$.

A. 0

B. 1

C. 2

D. 3

E. 4

3. Suppose that a spherical balloon is releasing its air inside at a rate of $3\pi \text{ cm}^3$ per second. What is the rate the radius of the balloon is decreasing when the balloon has a radius of 1 cm?

(Hint: The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$)

A. 0.25 cm/s

B. 0.5 cm/s

C. 0.75 cm/s

D. 1 cm/s

E. None of the above.

4. Let $g(x) = x^2 - x + 1$. Find the x -coordinates of the vertex of $g(x)$.

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. $-\frac{1}{2}$

E. $-\frac{1}{4}$

5. Let

$$f(x) = x^6 - x^5 + 100.$$

Find the interval for which $f(x)$ is concave (concave down).

A. $0 < x < \frac{2}{3}$

B. $x < 0$ or $x > \frac{2}{3}$

C. $x < \frac{5}{6}$

D. $x > \frac{5}{6}$

E. $0 < x < \frac{5}{6}$

6. Let $f(x) = \frac{x^2}{x^2-4}$. Find the interval for which the function is increasing.

A. $x > 0$

B. $x < 0$

C. $-2 < x < 2$

D. $0 < x < 2$

E. $-2 < x < 0$

7. Let

$$f(x) = \frac{x^2}{x^2 + 2x - 15}.$$

How many vertical asymptotes does $f(x)$ have?

A. 0

B. 1

C. 2

D. 3

E. 4

8. Let

$$f(x) = \frac{3\sqrt{x}}{\sqrt{x} + 2}.$$

Find the horizontal asymptotes of $f(x)$ as x tends to infinity.

A. $y = 0$

B. $x = 0$

C. $x + y = 3$

D. $y = 3$

E. $x = 3$

From questions 9 to 11. Please write down your solutions with steps in detail in the space provided.

9. Let

$$f(x) = x^3 - 9x^2 + 24x.$$

- (i) (2 points) Find $f'(x)$ and $f''(x)$.
- (ii) (2 points) Find the relative maximum and relative minimum of $f(x)$.
- (iii) (2 points) Find the point of inflection of $f(x)$.
- (iv) (2 points) Sketch $f(x)$. Indicate the points you found in (ii) and (iii).

10. (3 points) Let

$$(x^2 + y^2)^2 = 2xy.$$

Find $\frac{dy}{dx}$

11. (5 points) An ice block used in a cooler at a campsite is modelled as a cube of side length s . The block currently has volume $125\,000\text{ cm}^3$ and is melting at a rate of 1000 cm^3 per hour.

- (a) At what rate the side length is changing currently?
- (b) What is the rate of change of the total surface area?