Sketching Parabola.

Let $f(x) = ax^2 + bx + c$.

If $a > 0$, it opens up. If $a < 0$, it open down

Eg. 1. $f(x) = 2x^2 - 4x + 6$

$$= 2(x^2 - 2x + 3)$$

$$= 2\left(x^2 - 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + 3\right) \quad \text{(completing square)}$$

$$= 2\left((x-1)^2 - 1 + 3\right)$$

$$= 2\left((x-1)^2 + 2\right)$$

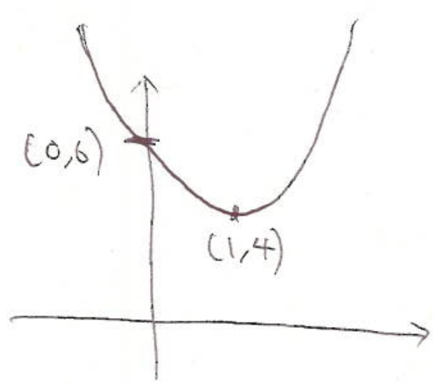
$$= 2(x-1)^2 + 4.$$

Vertex $= (1, 4)$

y-intercept $= f(0) = 6$.

No x-interept since $\Delta = (-4)^2 - 4(2)(6)$

$$= -32 < 0.$$



This vertex is the minimum point of $f(x)$.

Eg. 2.

$$\begin{aligned} f(x) &= -x^2+5x-6 \\ &= -\left(x^2-5x+6\right) \\ &= -\left(x^2-5x+\left(\tfrac{5}{2}\right)^2-\left(\tfrac{5}{2}\right)^2+6\right) \\ &= -\left(\left(x-\tfrac{5}{2}\right)^2-\tfrac{25}{4}+6\right) \\ &= -\left(\left(x-\tfrac{5}{2}\right)^2-\tfrac{1}{4}\right) \\ &= -\left(x-\tfrac{5}{2}\right)^2+\tfrac{1}{4} \end{aligned}$$

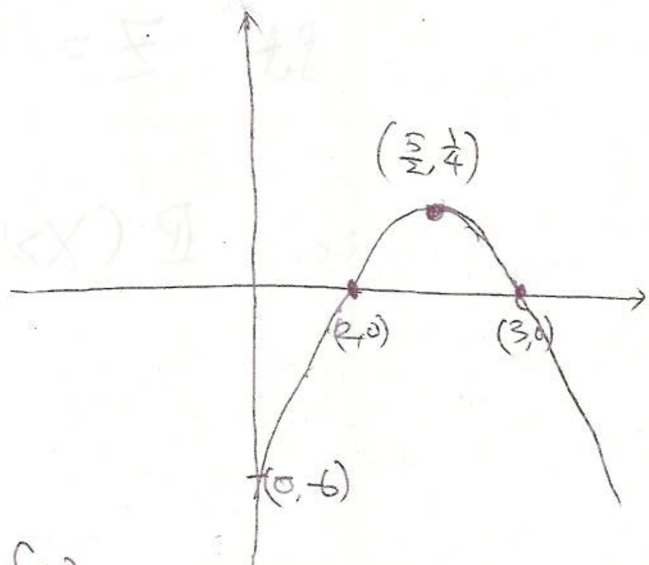
Vertex $= \left(\frac{5}{2}, \frac{1}{4}\right)$

y-intercept $= f(0) = -6$

x-intercept : Solving $f(x) = 0$

i.e. $-(x-2)(x-3) = 0$

$x = 2$ or 3.



This vertex is the maximum point of $f(x)$.

Techniques of completing Square.

$$x^2-bx+c = \underline{x^2-bx+\left(\tfrac{b}{2}\right)^2}-\left(\tfrac{b}{2}\right)^2+c$$

$$\downarrow$$

$$= \left(x-\tfrac{b}{2}\right)^2-\frac{b^2}{4}+c.$$