S4D03/S6D03 2019/2020: Assignment One

1. Construct an example showing the union of two σ-fields is not a σ-field. Verify your result.

2. Consider the sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$. Set $\mathcal{A} = \{\{1\}, \{3, 4\}, \{2, 4, 5\}\}$. Find the σ-field $\mathcal{F}$ generated by $\mathcal{A}$.

3. Let $E_1, E_2, \ldots$ be a sequence of disjoint measurable sets in the measurable space $(\Omega, \mathcal{F})$. Given a sequence of real numbers $a_1, a_2, \ldots$, define

$$f_n(\omega) = \sum_{i=1}^{n} a_i I_{E_i}(\omega)$$

and

$$f(\omega) = \sum_{i=1}^{\infty} a_i I_{E_i}(\omega).$$

Show that f_n converges pointwise to f as n tends to infinity.

4. Let Ω be the set of all rational numbers in $[0, 1]$. Set

$$\mathcal{C} = \{A_{a,b} : 0 \leq a \leq b \leq 1, A_{a,b} = \{\omega \in \Omega : a \leq \omega \leq b\}\}$$

and define the set function

$$\mu(A_{a,b}) = b - a.$$

Show that μ is not a probability.

Due date: 3:30pm September 19, 2019 in class.