S4D03/S6D03 2019/2020: Assignment Three

1. Let X_n be a binomial random variable with parameters n and p_n, and X be a Poisson random variable with parameter $\lambda > 0$. Assume that

$$\lim_{n \to \infty} np_n = \lambda.$$

Show that X_n converges in distribution to X as n tends to infinity.

2. Let X be a random variable with cumulative distribution function $F(\cdot)$ and

$$X_n = \frac{n}{n + \sqrt{n}} X.$$

Show that X_n converges to X in distribution.

3. Consider a sequence of non-negative random variables $\{Y_n : n \geq 1\}$ satisfying

$$\sum_{n=2}^{\infty} P\{Y_n > \log n\} < \infty.$$

Show that

$$\overline{\lim_{n \to \infty}} \frac{Y_n}{\log n} \leq 1.$$

4. Let $X_1, X_2, \ldots$ be iid exponential random variables with parameter $c > 0$. Set

$$M_n = \max\{X_1, \ldots, X_n\}, \quad b_n = c^{-1} \log n.$$

Let M be a random variable with cumulative distribution function $e^{-e^{-cx}}$. Show that $M_n - b_n$ converges to M in distribution as n tends to infinity.

Due date: 3:30pm on October 21, 2019.