S4D03/S6D03 2019/2020: Assignment Five

1. Let X be a Poisson random variable with parameter $\lambda > 0$. Show that $\frac{X-\lambda}{\sqrt{\lambda}}$ converges in distribution to the standard normal random variable Z as λ converges to infinity.

2. Let $\{X_n : n \geq 1\}$ be a sequence of independent random variables with

$$\mathbb{P}\{X_n = n\} = \mathbb{P}\{X_n = -n\} = \frac{1}{2n}, \ \mathbb{P}\{X_n = 0\} = 1 - \frac{1}{n}.$$

Set

$$S_n = \sum_{k=1}^{n} X_k, \ B_n^2 = \sum_{k=1}^{n} \mathrm{Var}[X_k].$$

Show that $\frac{S_n}{B_n}$ converges in distribution to a random variable W which has a characteristic function of the form

$$exp\{-\int_0^1 x^{-1}(1 - \cos xt)d\,x\}.$$

3. Assume that the random variables X and Y are independent, and $X+Y$ and X have the same distribution. Show that $Y = 0$ almost surely.

Due date: 3:30am in class on Nov.28, 2019.