A stock-flow consistent macroeconomic model for asset price bubbles

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Rational bubbles

- Consider a representative agent solving

$$\sup_c E_t \left[\sum_{j=1}^{\infty} \beta^{j-t} u(c_j) \right]$$

for exogenously given (e_t, d_t).

- The general solution for this problem is of the form $p_t = F_t + B_t$ where

$$F_t = \sum_{j=1}^{\infty} \beta^j E_t \left[d_{t+j} u'(e_{t+j} + d_{t+j}) \right]$$

is the fundamental price and B_t is a bubble term satisfying

$$E_t[B_{t+1}] = \beta^{-1} B_t \tag{1}$$

Consequences

- $B_t \geq 0$ for all t.
- Any nonzero rational bubble must start with $B_0 > 0$.
- If $T < \infty$, $B_t = 0$ for all $0 \leq t \leq T$, and this result is robust with respect to diverse information (Tirole 1982).
- If $T = \infty$, bubbles can exit in a myopic rational expectations equilibrium.
- Rational bubbles cannot exist in a fully dynamic REE with finitely many infinitely lived agents.
- They can exit in an overlapping generations models provided $0 < \bar{r} < g$, where $\bar{r}$ is the asymptotic real interest rate and g is the rate of growth of the economy (Tirole 1985).

Alternative models (Shiller, 1984)

- Consider a model where sophisticated investors have a demand function (portion of shares) of the form

$$Q_t^i = \frac{E_t[R_{t+1}] - \alpha}{\phi}. \tag{2}$$

- In addition, suppose there are noise traders who react to fads Y_t through a demand function $Q_t^n = Y_t/p_t$.
- In equilibrium we have $Q_t + \frac{Y_t}{p_t} = 1$.
- Inserting this into (2) and solving recursively leads to

$$p_t = \sum_{j=1}^{\infty} \frac{E_t[d_{t+j}] + \phi E_t[Y_{t-1+j}]}{(1+\alpha+\phi)^j}. \tag{3}$$

- This is also consistent with prices being not very forecastable.

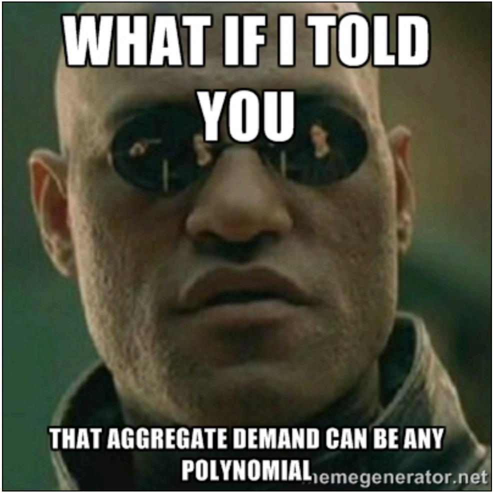
Other sources of inefficiencies

- Noise trader risk (DeLong, Shleifer, Summers and Waldmann 1990): prices deviate from fundamentals due to uncertainty created by noise traders, who can earn higher expected returns than sophisticated investors.
- Limits of arbitrage (Shleifer and Vishny 1997): fund managers leaving the market exactly when they are needed to restore fundamental value.
- No short-sales and diverse beliefs (Miller 1977, Harrison and Kreps 1978): pessimists stay on sidelines and optimists overbid
- Overconfidence (Scheinkman and Xiong 2003): mean reverting confidence levels lead to prices that contain an option to re-sell the asset at a later time.
- These are all microeconomic models. What about macro?

Dynamic Stochastic General Equilibrium (DSGE)

- Seeks to explain the aggregate economy using theories based on strong microeconomic foundations.
- Collective decisions of rational individuals over a range of variables for both present and future.
- All variables are assumed to be simultaneously in equilibrium.
- Equilibrium is only disrupted by exogenous shocks.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.
- Money is neutral in its effect on real variables.

SMD theorem: something is rotten in GE land



Stock-Flow Consistent models

- Stock-flow consistent models emerged in the last decade as a common language for many heterodox schools of thought in economics.
- They consider both real and monetary factors simultaneously.
- Specify the balance sheet and transactions between sectors.
- Accommodate a number of behavioural assumptions in a way that is consistent with the underlying accounting structure.
- Reject the RARE individual (representative agent with rational expectations) in favour of SAFE (sectoral average with flexible expectations) modelling.
- See Godley and Lavoie (2007) for the full framework.

Goodwin Model - SFC matrix

Balance Sheet	Households	Firms: current	Firms: capital	Sum
Capital			$+pK$	pK
Sum (net worth)	0	0	V_f	pK
Transactions				
Consumption	$-pC$	$+pC$		0
Investment		$+pI$	$-pI$	0
Acct memo [GDP]		$[pY]$		
Wages	$+W$	$-W$		0
Profits		$-\Pi$	$+\Pi_u$	0
Sum	0	0	0	0
Flow of Funds				
Capital			$+pI$	pI
Sum	0	0	Π_u	pI
Change in Net Worth	0	$pI + \dot{p}K - p\delta K$		$\dot{p}K + p\dot{K}$

Table: SFC table for the Goodwin model.

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}\ell}{pY} = \frac{\mathrm{w}}{pa} \quad \text{(wage share)}$$

$$\lambda = \frac{\ell}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

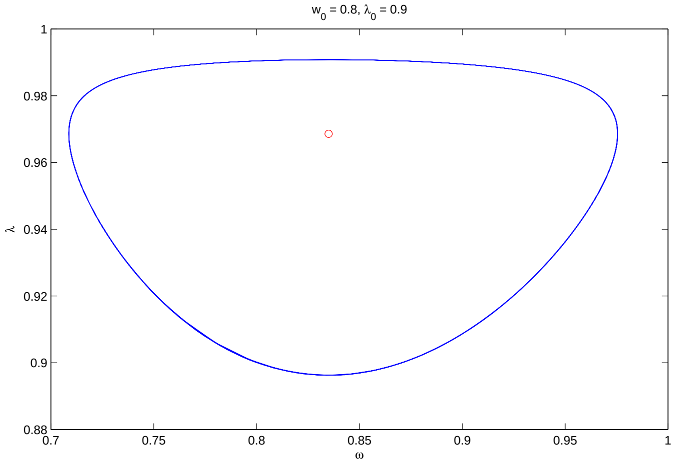
- It then follows that

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} - \frac{\dot{p}}{p} - \frac{\dot{a}}{a} = \Phi(\lambda, i, i^e) - i - \alpha$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\nu} - \alpha - \beta - \delta$$

- In the original model, all quantities were real (i.e divided by p), which is equivalent to setting $i = i^e = 0$.

Example 1: Goodwin model



Testing Goodwin on OECD countries

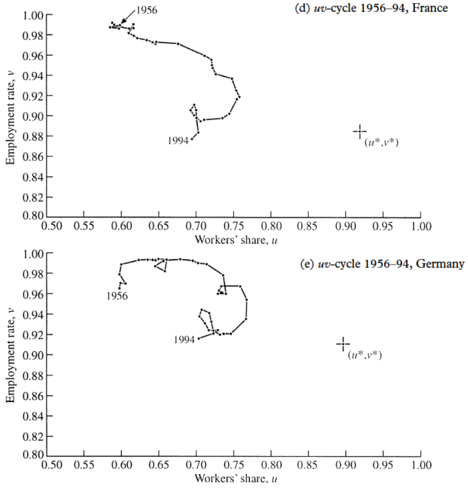


Figure: Harvie (2000)

Correcting Harvie (1970 to 2009)

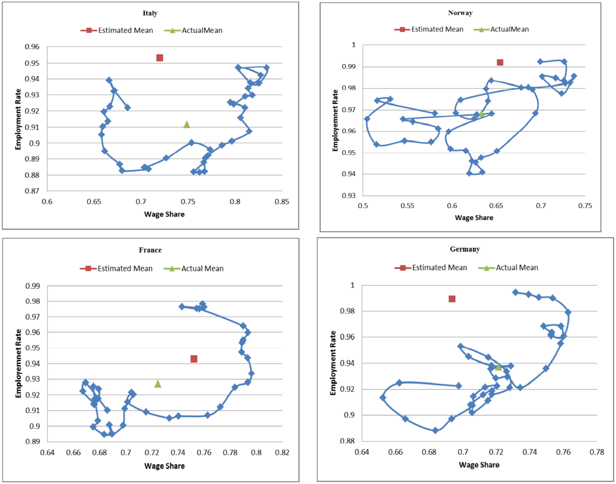


Figure: Grasselli and Maheshwari (2016, in progress)

SFC table for Keen (1995) model

Balance Sheet	Households	Firms current	Firms capital	Banks	Sum
Deposits	$+D$			$-D$	0
Loans			$-L$	$+L$	0
Capital			$+pK$		pK
Sum (net worth)	V_h	0	V_f	0	pK
Transactions					
Consumption	$-pC$	$+pC$			0
Investment		$+pI$	$-pI$		0
Acct memo [GDP]		$[pY]$			
Wages	$+W$	$-W$			0
Interest on deposits	$+rD$			$-rD$	0
Interest on loans		$-rL$		$+rL$	0
Profits		$-\Pi$	$+\Pi_u$		0
Sum	S_h	0	$S_f - pI$	0	0
Flow of Funds					
Deposits	$+\dot{D}$			$-\dot{D}$	0
Loans			$-\dot{L}$	$+\dot{L}$	0
Capital			$+pI$		pI
Sum	S_h	0	Π_u	0	pI
Change in Net Worth	S_h	$(S_f + \dot{p}K - p\delta K)$			$\dot{p}K + p\dot{K}$

Table: SFC table for the Keen model.

Keen model - Investment function

- Assume now that new investment is given by

$$\dot{K} = \kappa(\pi)Y - \delta K$$

where $\kappa(\cdot)$ is a nonlinear increasing function of profits $\pi = 1 - \omega - rd$.

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(\pi)Y - \pi Y$$

- The economy grows at a rate

$$g(\pi) := \frac{\dot{Y}}{Y} = \frac{\kappa(\pi)}{\nu} - \delta.$$

Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$
\begin{aligned}
\dot{\omega} &= \omega \left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda \left[g(\pi) - \alpha - \beta\right] \\
\dot{d} &= \kappa(\pi) - \pi - d g(\pi)
\end{aligned}
\tag{4}
$$

Example 3: convergence to the good equilibrium in a Keen model

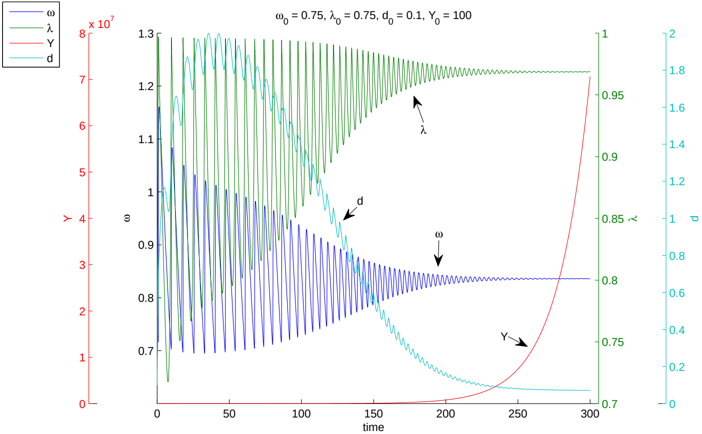


Figure: Grasselli and Costa Lima (2012)

Example 4: explosive debt in a Keen model

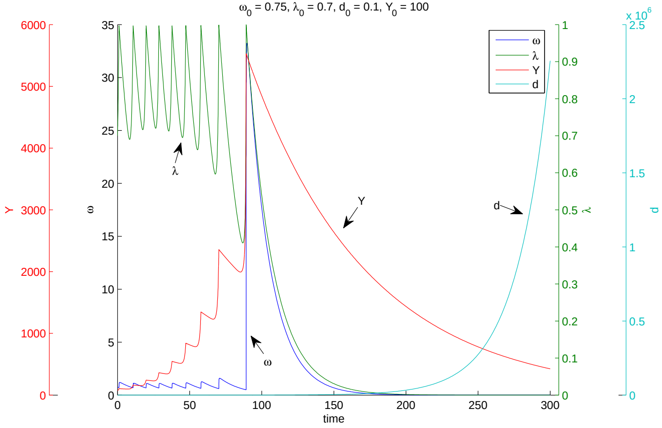
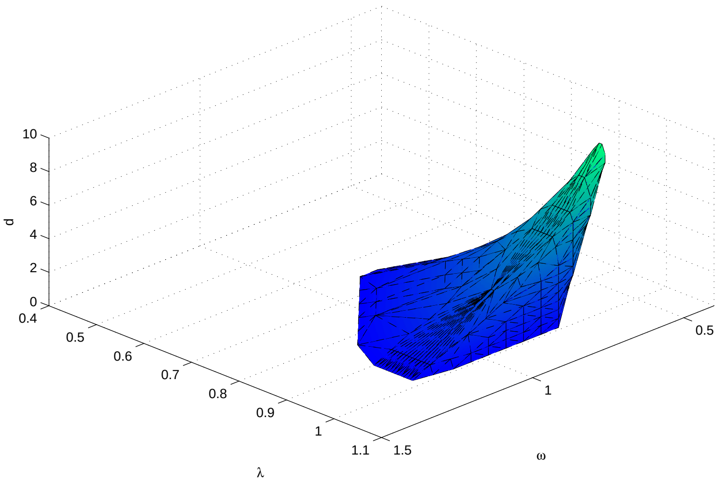
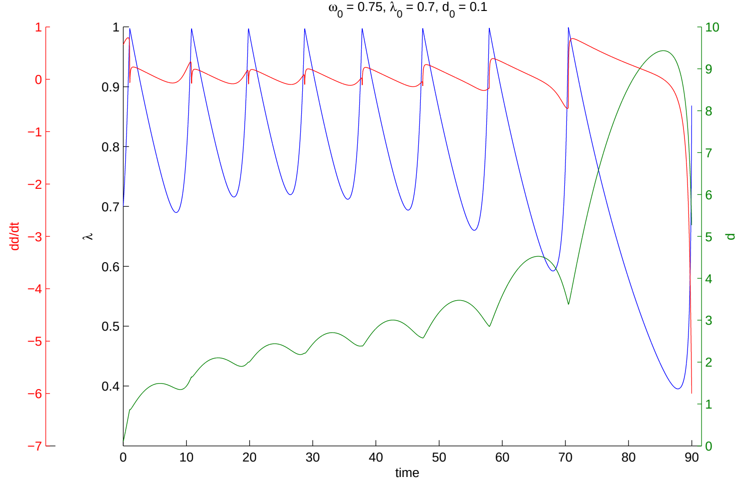


Figure: Grasselli and Costa Lima (2012)

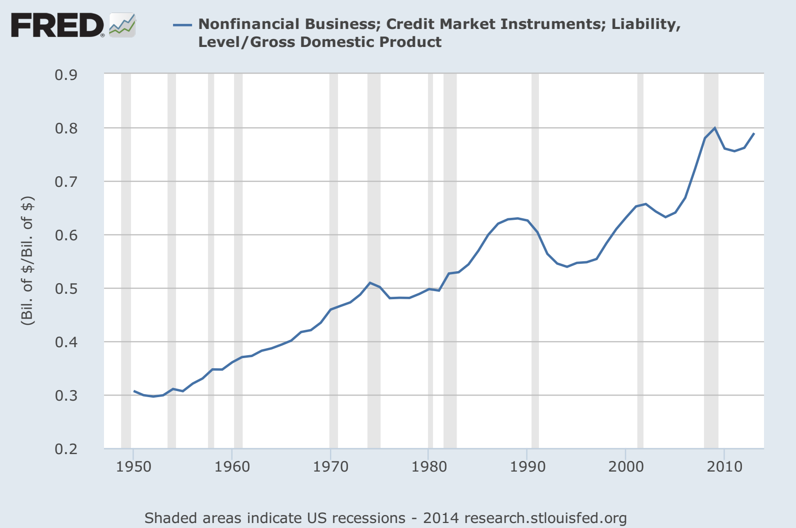
Basin of convergence for Keen model



Example 3 (continued): explosive debt in a Keen model



Corporate Debt share in the US 1950-2014



Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{L} = pI + r_L L - \kappa_L L + F$$
$$\dot{D}_f = pY - W + r_f D_f - \kappa_L L + F$$
$$\dot{F} = \Psi(g(\pi))F$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g(\omega, d) = \frac{\kappa(\pi)}{\nu} - \delta.$$

Ponzi financing - Differential equations

Setting $\kappa_L = r_L$ and defining $c = r_L b + (r_L - r_f)d_f$ and $f = F/(pY)$, where $d_f = D_f/(pY)$, the dynamical system becomes

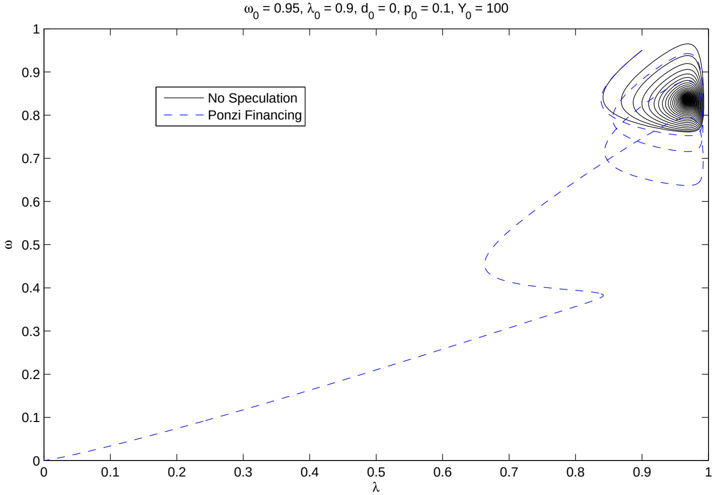
$$\dot{\omega} = \omega\left[\Phi(\lambda) - \alpha\right]$$

$$\dot{\lambda} = \lambda\left[g(\pi) - \alpha - \beta\right] \tag{5}$$

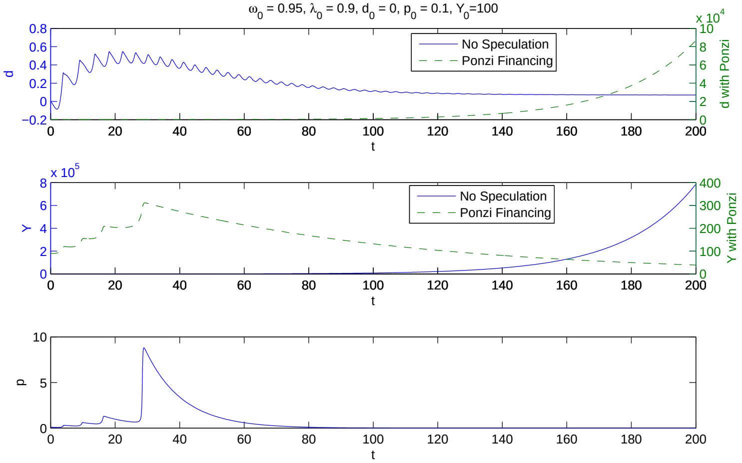
$$\dot{c} = r_L\kappa(\pi) - r_f\pi - c\left[g(\pi)\right] + (r_L - r_f)f \tag{6}$$

$$\dot{f} = f\left[\Psi\left(g(\pi)\right) - g(\pi)\right]$$

Example 4: effect of Ponzi financing



Example 4 (continued): effect of Ponzi financing



Credit and bubbles

- In *Manias, Panics, and Crashes*, Kindelberger and Aliber (2011) state that "most increases in the supply of credit do not lead to a mania - but nearly every mania has been associated with rapid growth in the supply of credit to a particular group of borrowers."
- Recall the Quantity Theory of Money equation

$$MV = pY, \tag{7}$$

where M is the money supply and V the velocity of circulation.
- In Werner (1997), this is replaced by

$$M_R V_R = pY \tag{8}$$
$$M_F V_F = SQ_F, \tag{9}$$

where R and F denote real and financial transactions respectively.

The monetary roots of bubbles and crashes

- In Corsi and Sornette (2012), this is model through

$$dM_t^F = \mu_F S_t M_t^F dt + \sigma_M M_t^F dW_t^F \tag{10}$$
$$dS_t = \mu_S M_t^F S_t dt + \sigma_S S_t dW_t^S, \tag{11}$$

which exhibits super-exponential behaviour.

- In our notation, the deterministic version of this model is

$$F = \frac{dM_F}{dt} = \mu_F S M_F \tag{12}$$
$$\frac{dS}{dt} = \mu_S M_F S \tag{13}$$

and exhibits finite-time singularity (FTS).

Stock price dynamics

- Instead of (10), we consider a stock price process of the form

$$\frac{dS_t}{S_{t_-}} = r_b dt + \sigma dW_t + j\mu_t dt - dJ_t$$

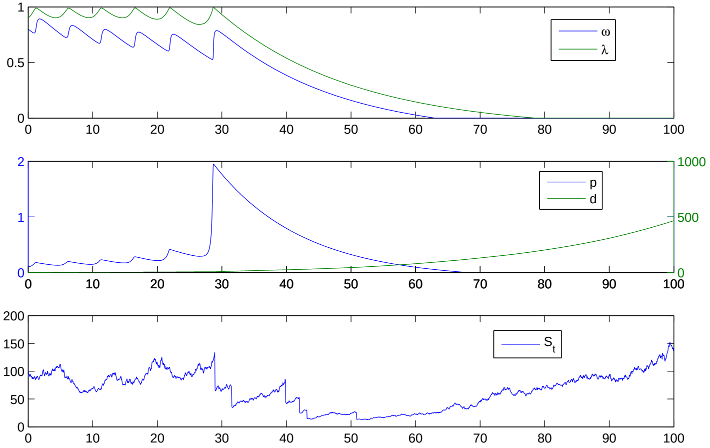
where J_t is an inhomogenous Poisson process with intensity $\mu_t = M(f(t))$ and jump sizes distributed on $(0, 1)$ with mean j.

- The interest rate for private debt is modelled as $r_t = r_b + r_p(t)$ where

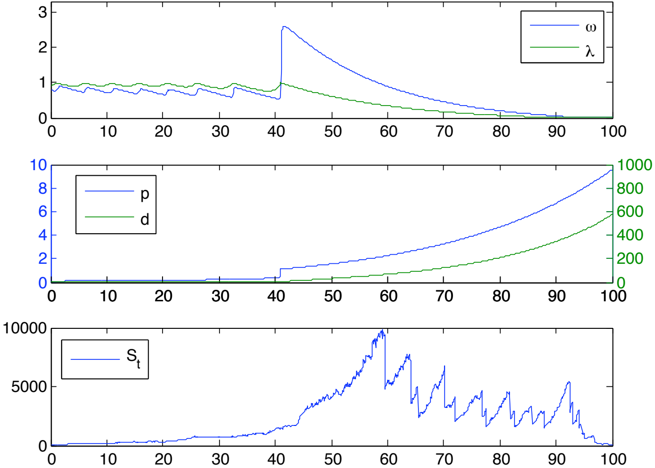
$$r_p(t) = \frac{\rho_1}{(S_t + \rho_2)^{\rho_3}}$$

for positive constants ρ_1, ρ_2, ρ_3.

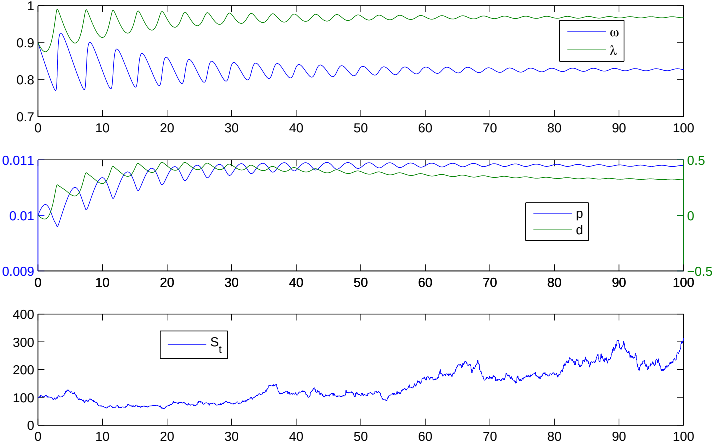
Example 5: stock prices, explosive debt, zero speculation



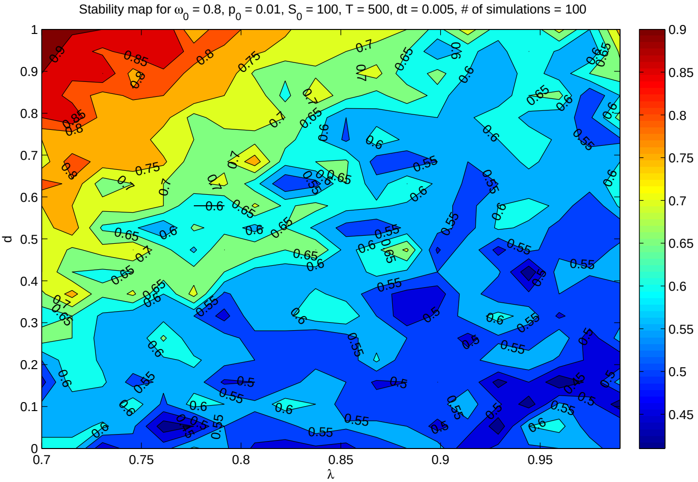
Example 6: stock prices, explosive debt, explosive speculation



Example 7: stock prices, finite debt, finite speculation



Stability map



Concluding remarks

- We provided a stock-flow consistent model for real-financial interactions as an extension of the Goodwin-Keen labour, investment, and debt dynamics.
- The modelling framework is an alternative to the dominant microfounded DSGE paradigm in macroeconomics.
- It incorporates insights from endogenous money theory, sectoral balances, and Minskian financial instability.
- Opens up new avenues for the application of modern dynamical systems techniques to economics.

Thank you!