Inventory growth cycles with debt-financed investment

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Inventory Cycles

- Small fraction of output (about 1% in the U.S.) but *major* fraction of changes in output (about 60% for postwar recession in the U.S.)

Table 1
Inventory Investment and Postwar Recessions

GNP Peak to Trough	*Change in[a] Real GNP*	*Change in[a] Inventory Investment*	*Change in Inventory Investment As a Percentage of Change in Real GNP*
1948 : 4–1949 : 4	−22.2	−28.2	127%
1953 : 2–1954 : 2	−43.7	−18.4	42%
1957 : 3–1958 : 1	−55.4	−21.7	39%
1960 : 1–1960 : 4	−17.5	−40.6	232%
1969 : 3–1970 : 4	−19.4	−28.2	145%[b]
1973 : 4–1975 : 1	−120.1	−78.1	65%
1980 : 1–1980 : 2	−76.4	−1.8	2%
1981 : 3–1982 : 3	−110.1	−45.1	41%[c]
			Average: 87%

[a]Billions of 1982 Dollars
[b]72% if trough is 1970 : 2
[c]90% if trough is 1982 : 4

Figure: Blinder and Mancini (1991)

Stylized facts

- Inventory investment is more volatile than output.
- Inventory investment is strongly countercyclical at very high frequencies (e.g., 2 - 3 quarters per cycle) but procyclical at business-cycle frequencies (e.g., 8 - 40 quarters per cycle).
- Production is less volatile than sales around the high frequencies; it is more volatile than sales only around business-cycle or lower frequencies.
- Most of the variance of inventory investment is concentrated around high frequencies rather than around business-cycle frequencies (unlike capital investment and GDP).

Theoretical models

- Micro theories view inventories primarily as a *stabilizing* factor (e.g production-smoothing).
- Incorporating inventories into fully micro-founded DSGE models is akin to incorporating money and finance.
- Earlier Keynesian model by Metzler (1941), further developed by Franke (1996) provides a more promising starting point.
- Heterodox (e.g stock-flow consistent) models emphasize the role of inventories, but fully developed models are rare and tend to be overcomplicated.

Contributions of this paper

- Combines the Franke (1996) model for inventory fluctuations with Goodwin (1967) model for labor market dynamics.
- Provides the first stock-flow consistent extension of the Keen (1996) model where both consumption and (debt-financed) investment are independently specified.
- Identifies and analyses two important sub-models: (i) the long-run model is a version of the Keen model with non-trivial effective demand, whereas (ii) the short-run model gives rise to Kitchin cycles (1923).

Notation

- Potential output: $Y_p = K/\nu$
- Actual output: $Y = Y_e + I_p$
- Capacity utilization: $u = Y/Y_p$
- Capital accumulation: $\dot{K} = I_k - \delta(u)K$
- Demand: $Y_d = C + I_k$
- Change in inventories: $\dot{V} = I_p + I_u = Y - Y_d$
- Unplanned changes: $I_u = Y - Y_d - I_p = Y_e - Y_d$.
- Gross investment: $I = Y - C = Y - Y_d + I_k = I_p + I_u + I_k$

Cost, prices, and financial balances

- Productivity: $a = Y/\ell$ (assume $\frac{\dot{a}}{a} = \alpha$)
- Employment rate: $\lambda = \ell/N = Y/(aN)$ (assume $\frac{\dot{N}}{N} = \beta$)
- Wage rate: $\mathrm{w} = W/\ell$
- Unit labour cost: $c = W/Y = \mathrm{w}/a$.
- Nominal output: $Y_n = pC + pI_k + c\dot{V}$.
- Profits: $\Pi = Y_n - W - rD - p\delta K$
- Change in debt for firms:

$$\dot{D} = p(I_k - \delta K) + c\dot{V} - \Pi = pI_k + c\dot{V} - \Pi_p,$$

where $\Pi_p = Y_n - W - rD$.

SFC Table

	Households	Firms		Banks	Sum
Balance Sheet					
Capital stock		$+pK$			$+pK$
Inventory		$+cV$			$+cV$
Deposits	$+M$			$-M$	0
Loans		$-D$		$+D$	0
Sum (net worth)	X_h	X_f		X_b	X
Transactions		current	capital		
Consumption	$-pC_h$	$+pC$		$-pC_b$	0
Capital Investment		$+pI_k$	$-pI_k$		0
Change in Inventory		$+c\dot{V}$	$-c\dot{V}$		0
Accounting memo [GDP]		$[Y_n]$			
Wages	$+W$	$-W$			0
Depreciation		$-p\delta K$	$+p\delta K$		0
Interest on deposits	$+r_m M$			$-r_m M$	0
Interest on loans		$-rD$		$+rD$	0
Profits		$-\Pi$	$+\Pi$		0
Financial Balances	S_h	0	$S_f - p(I_k - \delta K) - c\dot{V}$	S_b	0
Flow of Funds					
Change in Capital Stock		$+p(I_k - \delta K)$			$+p(I_k - \delta K)$
Change in Inventory		$+c\dot{V}$			$+c\dot{V}$
Change in Deposits	$+\dot{M}$			$-\dot{M}$	0
Change in Loans		$-\dot{D}$		$+\dot{D}$	0
Column sum	S_h	S_f		S_b	$p(I_k - \delta K) + c\dot{V}$
Change in net worth	$\dot{X}_h = S_h$	$\dot{X}_f = S_f + \dot{p}K + \dot{c}V$		$\dot{X}_b = S_b$	$\dot{X}$

Behavioural rules - firms

- Define

$$\pi_e = \frac{Y_{ne} - W - rD}{pY} = y_e(1-\omega) - rd,$$

where $y_e = Y_e/Y$, $\omega = W/(pY)$ and $d = D/(pY)$.

- We assume that sales expectations evolve as

$$\dot{Y}_e = g_e(u, \pi_e)Y_e + \eta_e(Y_d - Y_e)$$

- Let $V_d = f_d Y_e$ for a constant f_d and assume that

$$I_p = g_e(u, \pi_e)V_d + \eta_d(V_d - V).$$

- Moreover, take

$$I_k = \frac{\kappa(u, \pi_e)}{\nu}K.$$

Behavioural rules - banks and households

- We assume that
$$C = \theta(\omega, d)Y.$$
- This includes the case
$$\begin{aligned} pC_h &= c_{ih}[W + r_m M] + c_{wh}M, \\ pC_b &= c_{ib}[rD - r_m M] + c_{wb}(D - M). \end{aligned}$$
- In particular, we can have
$$pC = c_1 W + c_2 D \quad \Rightarrow \theta(\omega, d) = c_1\omega + c_2 d.$$
- Total demand is then given by
$$pY_d = pC + pI_k = p\theta(\omega, d)Y + p\frac{\kappa(u, \pi_e)}{\nu}K,$$
so that
$$y_d = \frac{Y_d}{Y} = \theta(\omega, d) + \frac{\kappa(u, \pi_e)}{u}.$$

Behavioural rules - wages and prices

- We assume that prices follow

$$\frac{\dot{p}}{p} = \eta_p \left(m\frac{c}{p} - 1 \right) - \eta_q \frac{Y_e - Y_d}{Y} = \eta_p \left(m\omega - 1 \right) + \eta_q (y_d - y_e) := i(\omega, y_d, y_e). \tag{1}$$

- The dynamics for nominal wages is

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \Phi(\lambda) + \gamma \frac{\dot{p}}{p}, \tag{2}$$

The main dynamical system

The full model is described by

$$
\begin{cases}
\dot{\omega} & = \omega\left[\Phi(\lambda) - \alpha - (1-\gamma) i(\omega, y_d, y_e)\right] \\
\dot{\lambda} & = \lambda\left[g(u, \pi_e, y_d, y_e) - \alpha - \beta\right] \\
\dot{d} & = d\left[r - g(u, \pi_e, y_d, y_e) - i(\omega, y_d, y_e)\right] + \omega - \theta(\omega, d) \\
\dot{y}_e & = y_e\left[g_e(u, \pi_e) - g(u, \pi_e, y_d, y_e)\right] + \eta_e(y_d - y_e) \\
\dot{u} & = u\left[g(u, \pi_e, y_d, y_e) - \frac{\kappa(u, \pi_e)}{\nu} + \delta(u)\right]
\end{cases}
$$

where

$$
i(\omega, y_d, y_e) = \eta_p\,(m\omega - 1) + \eta_q(y_d - y_e)
$$

and

$$
\begin{aligned}
g(u, \pi_e, y_d, y_e) = \left[f_d(g_e(u, \pi_e) + \eta_d) + 1\right]&\big(y_e g_e(u, \pi_e) \\
&+ \eta_e(y_d - y_e)\big) + \eta_d(y_d - 1)
\end{aligned}
$$

Interior equilibrium

- It follows from the second equation that

$$g(\overline{u}, \overline{\pi}_e, \overline{y}_d, \overline{y}_e) = \alpha + \beta.$$

- Inserting this in the fourth equation gives $\overline{y}_d = \overline{y}_e$ and

$$g_e(\overline{u}, \overline{\pi}_e) = \alpha + \beta.$$

- Substitution in the definition of g then gives

$$\overline{y}_d = \overline{y}_e = \frac{1}{1 + (\alpha + \beta) f_d}.$$

- Moreover, it follows that $\overline{v} = f_d \overline{y}_e$, so that the equilibrium level of inventory is the desired level $V_d = f_d \overline{y}_e Y$.
- Furthermore, we see from the definition of i that

$$i(\overline{\omega}, \overline{y}_d, \overline{y}_e) = i(\overline{\omega}) = \eta_p(m\overline{\omega} - 1).$$

Interior equilibrium - continued

- From the third equation, we see that:

$$\overline{d} = \frac{\overline{\omega} - \theta(\overline{\omega}, \overline{d})}{\alpha + \beta + i(\overline{\omega}) - r}. \tag{3}$$

- From the last equation, we obtain:

$$\kappa(\overline{\pi}_e, \overline{u}) = \nu[\alpha + \beta + \delta(\overline{u})], \tag{4}$$

which can be inserted in the demand function to give

$$\overline{u} = \frac{\nu[\alpha + \beta + \delta(\overline{u})](1 + (\alpha + \beta)f_d)}{1 - \theta(\overline{\omega}, \overline{d})(1 + (\alpha + \beta)f_d)}.$$

- We can then obtain the values of $(\overline{\omega}, \overline{d})$ by solving (3)-(4).
- Finally, the first equation gives

$$\Phi(\overline{\lambda}) = \alpha + (1 - \gamma)i(\overline{\omega}).$$

Goodwin model

- Model in real terms: $\eta_p = \eta_q = \gamma = 0$, $p = 1$.
- No inventories: $f_d = \eta_d = V_d = I_p = 0$
- Output equals demand: $\eta_e \to \infty$, $Y_e = Y_d = Y$
- Constant capital-to-output ratio: $u = 1$.
- Constant depreciation: $\delta(u) = \delta > 0$.
- Investment equals profits: $\kappa(u, \pi_e) = \pi_e = 1 - \omega - rd$.
- No banks: $\dot{D} = 0$, take $d = D_0 = 0$.
- All wages are consumed: $c_{ih} = c_1 = 1$ (and $c_2 = r$).
- This leads to

$$\begin{cases} \dot{\omega} & = \omega\left[\Phi(\lambda) - \alpha\right] \\ \dot{\lambda} & = \lambda\left[\frac{1-\omega}{\nu} - \alpha - \beta - \delta\right], \end{cases} \tag{5}$$

Franke model

- Model in real terms: $\eta_p = \eta_q = \gamma = 0$ and $p = 1$
- Variables normalized by K instead of Y, resulting in the intensive variables: $u^F := Y/K = u/\nu$, $z^F := Y_e/K = y_e u^F$, $v^F =: V/K = vu^F$.
- Constant wage share ω: $\dot{\omega} = 0$.
- Second equation in our system decouples.
- No banks: $\dot{d} = 0$.
- Constant long-run expected growth: $g_e(u, \pi_e) = \alpha + \beta$.
- Investment as function of utilization: $\kappa(u, \pi_e) = \nu h(u^F)$.
- Excess demand as a function of u^F: $y^d = e(u^F) + 1$.
- We then obtain the same system as in Franke (1996) from our fourth and fifth equations, leading to

$$v^F = \frac{f_d \overline{u}^F}{1 + (\alpha+\beta) f_d} = \overline{v}\,\overline{u}^F, \quad \overline{z}^F = \frac{\overline{u}^F}{1 + (\alpha+\beta) f_d} = \overline{y}_e \overline{u}^F.$$

Keen model

- Model in real terms: $\eta_p = \eta_q = \gamma = 0$ and $p = 1$
- Same as Goodwin for production and inventories: $f_d = \eta_d = V_d = I_p = 0$, $\eta_e \to \infty$, $Y_e = Y_d = Y$, $u = 1$, $\delta(u) = \delta$.
- Investment as function of profits: is now given by $\kappa(u, \pi_e) = \kappa(\pi_e) = \kappa(1 - \omega - rd)$.
- Accommodating consumption: $C = Y_d - I_k = (1 - \kappa(\pi_e))Y$, $\theta(\omega, d) = 1 - \kappa(1 - \omega - rd)$.
- With these parameter choices, the system reduces to

$$
\begin{cases}
\dot{\omega} & = \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} & = \lambda\left[\frac{\kappa(\pi_e)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} & = d\left[r - \frac{\kappa(\pi_e)}{\nu} - \delta\right] + \omega - 1 + \kappa(\pi_e)
\end{cases}
$$

Monetary Keen model

- As shown in Grasselli Nguyen-Huu (2015), it is easy to incorporate inflation in the original Keen model.
- Adopting all the parameter choices and functional forms of the previous section (including $\eta_q = 0$) with the exception of arbitrary constants η_p and γ, we find

$$\begin{cases} \dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha - (1-\gamma)i(\omega)\right] \\ \dot{\lambda} &= \lambda\left[\frac{\kappa(\pi_e)}{\nu} - \alpha - \beta - \delta\right] \\ \dot{d} &= d\left[r - \frac{\kappa(\pi_e)}{\nu} - \delta - i(\omega)\right] + \omega - 1 + \kappa(\pi_e) \end{cases} \tag{6}$$

  where $\pi_e = 1 - \omega - rd$ and $i(\omega) = \eta_p(m\omega - 1)$.
- Apart from the usual "good" and "bad" equilibrium, this system also admits a new class equilibria of the form $(\overline{\omega}_3, 0, \overline{d}_3)$ or $(\overline{\omega}_3, 0, +\infty)$ where

$$\overline{\omega}_3 = \frac{1}{m} + \frac{\Phi(0) - \alpha}{m\eta_p(1-\gamma)}, \quad i(\overline{\omega}_3) < 0.$$

Long-run dynamics

- Take $\eta_e = \eta_d = f_d = 0$ so that $Y = Y_e$.
- We then have $g(u, \pi_e, y_d, y_e) = g_e(u, \pi_e)$.
- The system then becomes

$$\begin{cases} \dot{\omega} &= \omega \left[\Phi(\lambda) - \alpha - (1-\gamma) i(\omega, y_d)\right] \\ \dot{\lambda} &= \lambda \left[g_e(u, \pi_e) - \alpha - \beta\right] \\ \dot{d} &= d\left[r - g_e(u, \pi_e) - i(\omega, y_d)\right] + \omega - \theta(\omega, d) \\ \dot{u} &= u\left[g_e(u, \pi_e) - \frac{\kappa(u, \pi_e)}{\nu} + \delta(u)\right], \end{cases}$$

  where $\pi_e = 1 - \omega - rd$ and
  $i(\omega, y_d) = \eta_p(m\omega - 1) + \eta_q(y_d - 1)$.
- In the special case $g_e(u, \pi_e) = \alpha + \beta$ (as in the Franke model), we have $\dot{\lambda} = 0$, so the interior equilibrium can only be achieved if $\lambda_0 = \Phi^{-1}(\alpha + (1-\gamma)\overline{\omega})$.

Keen model with inventories - real version

- Take $\eta_e = \eta_d = f_d = 0$ so that $Y = Y_e$ as in the long-run dynamics above, so that $g(u, \pi_e, y_d, y_e) = g_e(u, \pi_e)$.
- In addition, consider the model in real terms, that is $\eta_p = \eta_q = \gamma = 0$ and $p = 1$.
- Setting $g_e(u, \pi_e) = \frac{\kappa(u,\pi_e)}{\nu} - \delta(u)$ leads to

$$\begin{cases} \dot{\omega} & = \omega\left[\Phi(\lambda) - \alpha\right] \\ \dot{\lambda} & = \lambda\left[\frac{\kappa(u_0,\pi_e)}{\nu} - \delta(u_0) - \alpha - \beta\right] \\ \dot{d} & = d\left[r - \frac{\kappa(u_0,\pi_e)}{\nu} + \delta(u_0)\right] + (1 - c_1)\omega - c_2 d, \end{cases}$$

where we took $\theta(\omega, d) = c_1\omega + c_2 d$.
- This is the closest model to the original Keen model but with $y_d = c_1\omega + c_2 d + \frac{\kappa(u_0,\pi_e)}{u_0}$ and

$$\dot{\mathrm{v}} = \left(1 - c_1\omega - c_2 d - \frac{\kappa(u_0,\pi_e)}{u_0}\right) - \left(\frac{\kappa(u_0,\pi_e)}{\nu} - \delta(u_0)\right)\mathrm{v}.$$

Keen model with inventories - monetary version

- Using (1)-(2) as the price-wage dynamics leads to the following monetary version of the model of the previous section

$$\begin{cases} \dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha - (1-\gamma)i\right] \\ \dot{\lambda} &= \lambda\left[\frac{\kappa(u_0,\pi_e)}{\nu} - \delta(u_0) - \alpha - \beta\right] \\ \dot{d} &= d\left[r - \frac{\kappa(u_0,\pi_e)}{\nu} + \delta(u_0) - i\right] + (1-c_1)\omega - c_2 d \end{cases}$$

where

$$i(\omega, d) = \eta_p\left(m\omega - 1\right) + \eta_q(y_d - 1)$$

- As before, we regard this as the closest model to the monetary Keen model in (6), but with a non-trivial effective demand and fluctuating inventory levels.

Short-run dynamics

- Suppose now that $\alpha + \beta = 0$, $g_e(u, \pi_e) = 0$ (no growth)
- Assume further that $\kappa(u, \pi_e) = \nu\delta(u)$.
- This leads to

$$
\begin{aligned}
v &= \frac{[1 + f_d\eta_d]y_e - 1}{\eta_d}, \\
g(y_e, y_d) &= \eta_e(1 + f_d\eta_d)(y_d - y_e) + \eta_d(y_d - 1),
\end{aligned}
$$

and the main system reduces to

$$
\begin{cases}
\dot{\omega} = \omega[\Phi(\lambda) - (1 - \gamma)i(\omega, y_d, y_e)] \\
\dot{\lambda} = \lambda g(y_e, y_d) \\
\dot{d} = d[r - g(y_e, y_d) - i(\omega, y_d, y_e)] + \omega - \theta(\omega, d) \\
\dot{y}_e = -y_e g(y_e, y_d) + \eta_e(y_d - y_e) \\
\dot{u} = ug(y_e, y_d)
\end{cases}
$$

Planar dynamics

- Assume now that $\eta_p = 0$ and $\Phi(\cdot) \equiv 0$, so that

$$i(\omega, y_d, y_e) = i(y_d, y_e) = \eta_q(y_d - y_e).$$

- Moreover, let $\delta(u) = \delta u$ for $\delta > 0$ and

$$\theta(\omega, d) = c_1\omega + c_2 d = c_1\omega \qquad (\text{i.e. } \ c_2 = 0) \tag{7}$$

- This gives $y_d = c_1\omega + \nu\delta$ so the system decouples and we can focus on

$$\begin{cases} \dot{y}_d & = -(1-\gamma)y_d\eta_q(y_d - y_e) \\ \dot{y}_e & = \eta_e(y_d - y_e) - y_e g(y_e, y_d) \end{cases} \tag{8}$$

with (ω, λ, d) satisfying a subordinated system that can be solved after.

Equilibrium analysis

- The previous system admits the equilibria $(1,1)$, $(0,0)$ and $(+\infty,+\infty)$.
- The equilibrium $(1,1)$ is locally stable provided $\gamma < \gamma_0 := 1 - \eta_e\eta_d f_d/\eta_q$.
- At $\gamma = \gamma_0$ there is a sub-critical Andronov-Hopf bifurcation and for $\gamma \geq \gamma_0$ the equilibrium is unstable.
- The equilibrium $(0,0)$ is unstable provided $\eta_d > \eta_e$ and fails to be asymptotically stable, even if $\eta_d < \eta_e$.
- The equilibrium $(+\infty,+\infty)$ is characterized by a finite-time blow-up with $y_d/y_e \to 0$ for a large set of initial conditions.

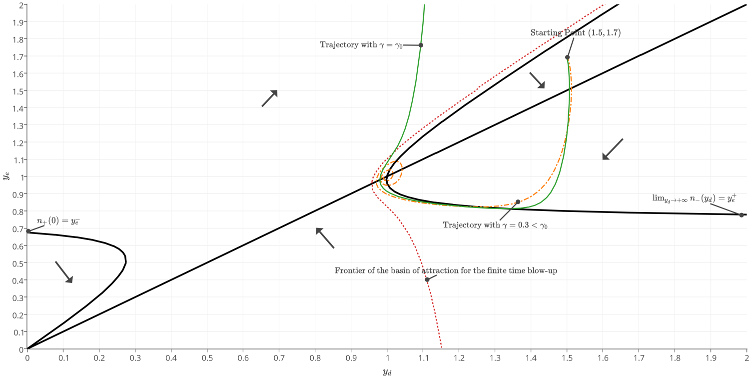

Figure: Short-run dynamics with $i(\omega, y_d, y_e) = i(y_d, y_e) = \eta_q(y_d - y_e)$

Kitchin cycles

- Consider now

$$\frac{\dot{p}}{p} = \eta_p \left(m\frac{c}{p} - 1 \right) - \eta_q \frac{V_d - V}{Y}$$

which, along with previous assumptions, provides the inflation rate $i(y_e) = \eta_q (1 - y_e)/\eta_v$.

- This now leads to

$$\begin{cases} \dot{y}_d &= -(1-\gamma) y_d \frac{\eta_q}{\eta_v}(1 - y_e) \\ \dot{y}_e &= \eta_e (y_d - y_e) - y_e g(y_e, y_d), \end{cases} \tag{9}$$

- The slight difference with the latter concerns the first equation, for which the isocline is given by $\{y_e = 1\}$ instead of $\{y_d = y_e\}$.

Equilibrium analysis

- The new system also admits the equilibria $(1,1)$, $(0,0)$ and $(+\infty,+\infty)$.
- The equilibrium $(1,1)$ is now locally unstable for all parameters.
- On the other hand, the equilibrium $(0,0)$ is locally stable provided $\eta_d < \eta_e$.
- The finite-time blow-up is similar to the previous case.

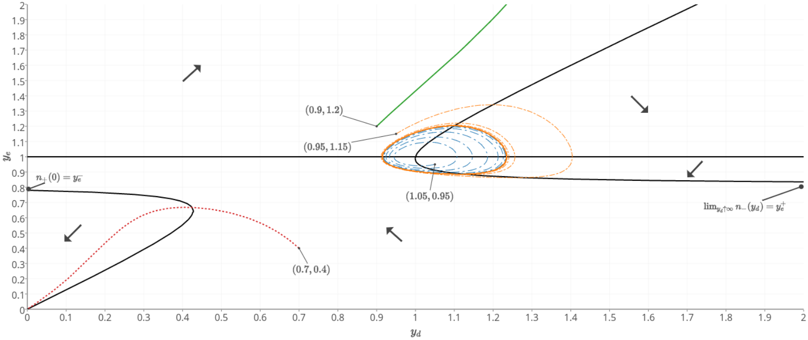

Figure: Short-run dynamics with
$i(\omega, y_d, y_e) = i(y_d, y_e) = \eta_q(1 - y_e)/\eta_v$

Concluding remarks

- We have introduced a stock-flow consistent model for inventory growth cycles with debt-financed investment.
- The model unifies features of several simpler models previously proposed in the heterodox economics literature (Goodwin, Franke, Keen).
- We identified the interior equilibrium of the full model and analyzed in detail the stability of two classes of sub-models.
- The long-run dynamics arising from ignoring short-run fluctuations can be regarded as a Keen model with inventories.
- The short-run dynamics arising solely from tracking inventory fluctuations in an imperfect information setting can be regarded as a formalization of Kitchin cycles.

Thank you!