A dynamical systems model for credit expansion, asset price bubbles and financial fragility

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Outline

Dynamic General Equilibrium views

- Seeks to explain the aggregate economy using theories based on strong microeconomic foundations.
- Collective decisions of rational individuals over a range of variables for both present and future.
- All variables are assumed to be simultaneously in equilibrium.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.
- Money is neutral in its effect on real variables.
- Largely ignores uncertainty by simply subtracting risk premia from all risky returns and treat them as risk-free.

Voices of discontent

- M. Morishima (1984): "If economists successfully devise a correct general equilibrium model (...) should it lack the institutional backing to realize an equilibrium solution, then [it] will amount to no more than a utopian state of affairs which bears no relation whatsoever to the real economy."
- A. Kirman (1989): "[DSGE is] empty in the sense that one cannot expect it to house the elements of a scientific theory, one capable of producing empirically falsifiable propositions".
- K. Arrow (1986): "In the aggregate, the hypothesis of rational behavior has in general no implications."
- R. Solow (2006): "Maybe there is in human nature a deep-seated perverse pleasure in adopting and defending a wholly counterintuitive doctrine that leaves the uninitiated peasant wondering what planet he or she is on."

Minsky's alternative interpretation of Keynes

- Neoclassical economics is based on barter paradigm: money is convenient to eliminate the double coincidence of wants.
- In a modern economy, firms make complex portfolios decisions: which assets to hold and how to fund them.
- Financial institutions determine the way funds are available for ownership of capital and production.
- Uncertainty in valuation of cash flows (assets) and credit risk (liabilities) drive fluctuations in real demand and investment.
- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.

Minsky's Financial Instability Hypothesis

- Start when the economy is doing well but firms and banks are conservative.
- Most projects succeed - "Existing debt is easily validated: it pays to lever".
- Revised valuation of cash flows, exponential growth in credit, investment and asset prices.
- Highly liquid, low-yielding financial instruments are devalued, rise in corresponding interest rate.
- Beginning of "euphoric economy": increased debt to equity ratios, development of Ponzi financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
- Euphoria becomes a panic.
- "Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing".

Goodwin Model (1967) - Assumptions

- Assume that

$$
\begin{aligned}
N(t) &= N_0 e^{\beta t} && \text{(total labour force)} \\
a(t) &= a_0 e^{\alpha t} && \text{(productivity per worker)} \\
Y(t) &= \nu K(t) = a(t)L(t) && \text{(total yearly output)}
\end{aligned}
$$

  where K is the total stock of capital and L is the employed population.

- Assume further that

$$
\begin{aligned}
\dot{\mathrm{w}} &= \Phi(\lambda)\mathrm{w} && \text{(Phillips curve)} \\
\dot{K} &= (Y - \mathrm{w}L) - \delta K && \text{(Say's Law)}
\end{aligned}
$$

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}L}{Y} = \frac{\mathrm{w}}{a} \quad \text{(wage share)}$$

$$\lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

- It then follows that

$$\dot{\omega} = \omega(\Phi(\lambda) - \alpha)$$

$$\dot{\lambda} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \beta - \delta\right)$$

Goodwin Model - Properties

- If we take Φ to be linear, this is a predator-prey model.
- To ensure $\lambda \in (0,1)$ we assume instead that Φ is $C^1(0,1)$ and satisfies

$$\Phi'(\lambda) > 0 \text{ on } (0,1)$$
$$\Phi(0) < \alpha$$
$$\lim_{\lambda \to 1^-} \Phi(\lambda) = \infty.$$

- Then $(\overline{\omega}_0, \overline{\lambda}_0) = (0,0)$ is a saddle point and the only other equilibrium

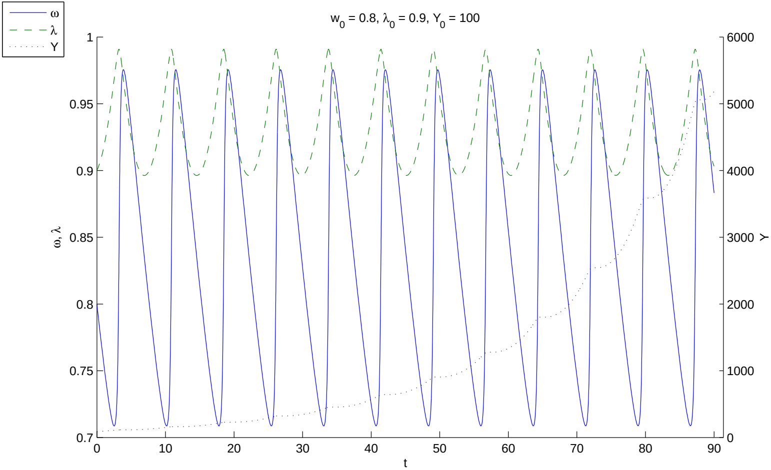
$$(\overline{\omega}_1, \overline{\lambda}_1) = \left(1 - \nu(\alpha + \beta + \delta), \Phi^{-1}(\alpha)\right)$$

is non-hyperbolic.
- Moreover

$$g(\overline{\omega}_1) := \frac{\dot{Y}}{Y}(\overline{\omega}_1) = \frac{1 - \overline{\omega}_1}{\nu} - \delta = \alpha + \beta,$$

Example 1: Goodwin model

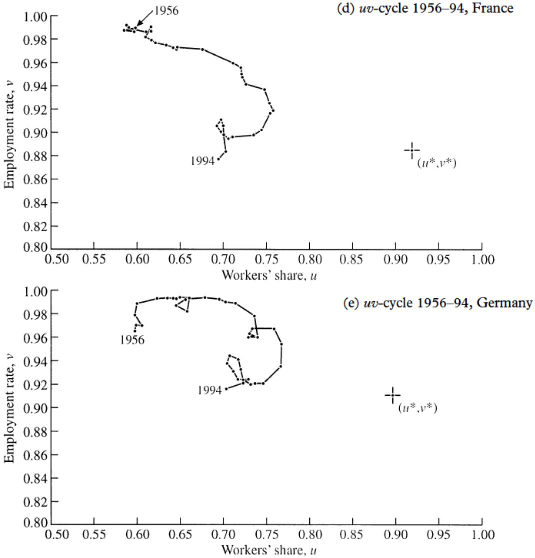
Example 1 (continued): Goodwin model



Goodwin Model - Extensions, structural instability, and empirical tests

- Desai 1972: Inflation leads to a stable equilibrium.
- Ploeg 1985: CES production function leads to stable equilibrium.
- Goodwin 1991: Pro-cyclical productivity growth leads to explosive oscillations.
- Solow 1990: US post-war data shows three sub-cycles with a "bare hint of a single large clockwise sweep" in the (ω, λ) plot.
- Harview 2000: Data from other OECD confirms the same qualitative features and shows unsatisfactory quantitative estimations.

Testing Goodwin on OECD countries



Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

where $\kappa(\cdot)$ is $C^1(-\infty, \infty)$ satisfying

$$\kappa'(\pi) > 0 \text{ on } (-\infty, \infty)$$

$$\lim_{\pi \to -\infty} \kappa(\pi) = \kappa_0 < \nu(\alpha + \beta + \delta) < \lim_{\pi \to +\infty} \kappa(\pi)$$

$$\lim_{\pi \to -\infty} \pi^2 \kappa'(\pi) = 0.$$

Accordingly, total output evolves as

$$\frac{\dot{Y}}{Y} = \frac{\kappa(1 - \omega - rd)}{\nu} - \delta := g(\omega, d)$$

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y$$

Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned} \tag{1}$$

Good equilibrium

- Define

$$\overline{\pi}_1 = \kappa^{-1}(\nu(\alpha+\beta+\delta))$$

- We verify that

$$\overline{\omega}_1 = 1 - \overline{\pi}_1 - r\frac{\nu(\alpha+\beta+\delta) - \overline{\pi}_1}{\alpha+\beta}$$

$$\overline{\lambda}_1 = \Phi^{-1}(\alpha)$$

$$\overline{d}_1 = \frac{\nu(\alpha+\beta+\delta) - \overline{\pi}_1}{\alpha+\beta}$$

is an equilibrium for (1) and satisfies the relation

$$1 - \overline{\omega}_1 - r\overline{d}_1 = \overline{\pi}_1$$

- Moreover

$$g(\overline{\omega}_1, \overline{d}_1) = \frac{\kappa(1 - \overline{\omega}_1 - r\overline{d}_1)}{\nu} - \delta = \alpha + \beta.$$

Keen model - Irrelevant equilibria

- Other equilibrium points are given by

$$(\overline{\omega}_0, \overline{\lambda}_0, \overline{d}_0) = (0, 0, \overline{d}_0) \tag{2}$$

where $\overline{d}_0$ is any solution of the equation

$$d\left[r - \frac{\kappa(1-rd)}{\nu} + \delta\right] + \kappa(1-rd) - 1 = 0$$

- Another set of equilibrium points are $(0, \lambda, \overline{d}_1)$ provided $1 - r\overline{d}_1 = \overline{\pi}_1$, that is

$$1 - r\frac{\nu(\alpha+\beta+\delta) - \kappa^{-1}(\nu(\alpha+\beta+\delta))}{\alpha+\beta} = \kappa^{-1}(\nu(\alpha+\beta+\delta)).$$

Keen model - Explosive debt

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

$$\begin{aligned}
\dot{\omega} &= \omega \left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda \left[\frac{\kappa(1-\omega-r/u)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{u} &= u \left[\frac{\kappa(1-\omega-r/u)}{\nu} - r - \delta\right] - u^2 \left[\kappa(1-\omega-r/u) - (1-\omega)\right].
\end{aligned} \tag{3}$$

- We now see that $(0, 0, 0)$ is an equilibrium of (3) corresponding to the point

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2) = (0, 0, +\infty)$$

for the original system.

Keen model - Local stability

- Analyzing the Jacobian of (1) and (3) we obtain the following conclusions.
- The good equilibrium $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1)$ is stable if and only if

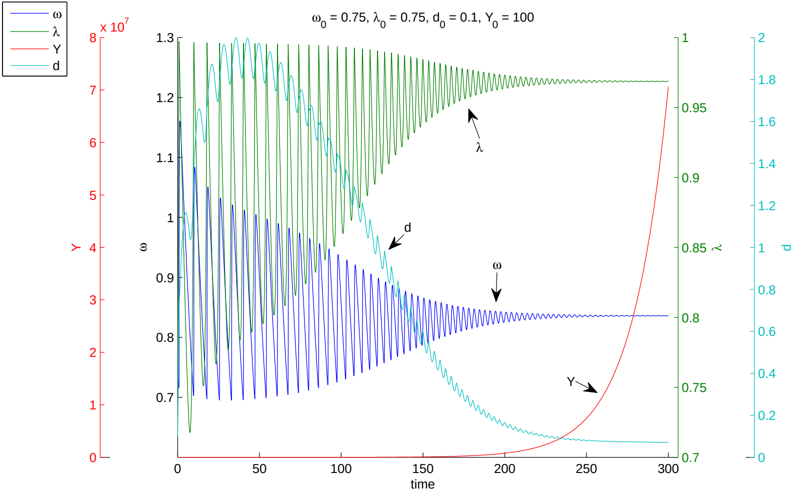
$$r\left[\frac{\kappa'(\overline{\pi}_1)}{\nu}\left(\overline{\pi}_1 - \kappa(\overline{\pi}_1) + \nu(\alpha+\beta)\right) - (\alpha+\beta)\right] > 0.$$

- The equilibrium $(0, 0, \overline{d}_0)$ is typically a saddle point.
- The equilibria $(0, \lambda, \overline{d}_1)$ are structurally unstable.
- The point $(0, 0, 0)$ is a stable equilibrium for (3) if and only if

$$\frac{\kappa_0}{\nu} - \delta < r.$$

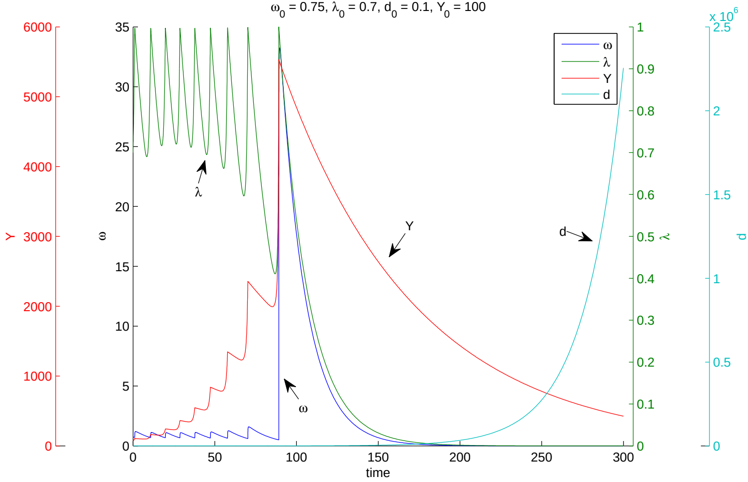
Example 2 : convergence to the good equilibrium in a Keen model

Example 2 (continued): convergence to the good equilibrium in a Keen model



Example 3: explosive debt in a Keen model

Example 3 (continued): explosive debt in a Keen model



Example 3 (continued): explosive debt in a Keen model

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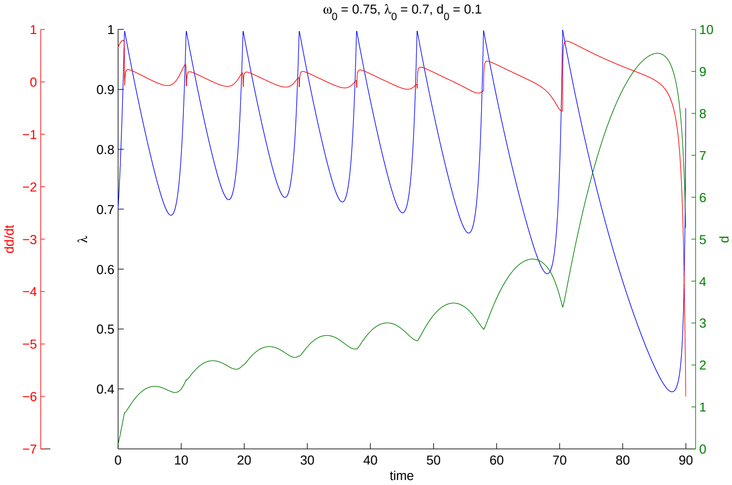
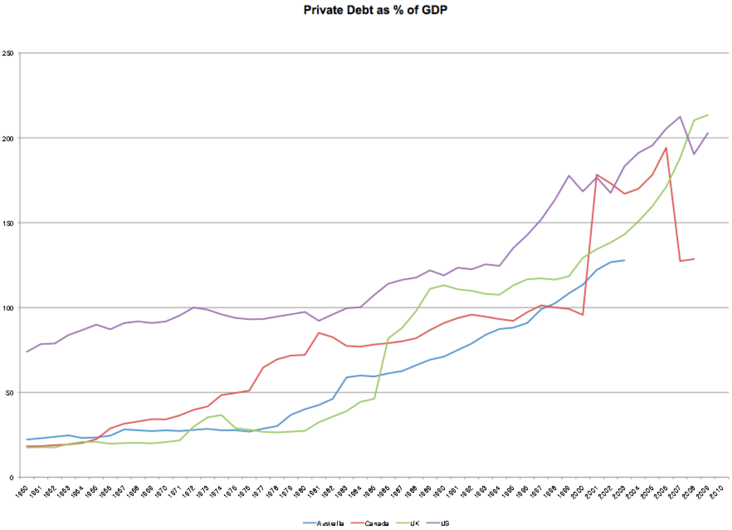
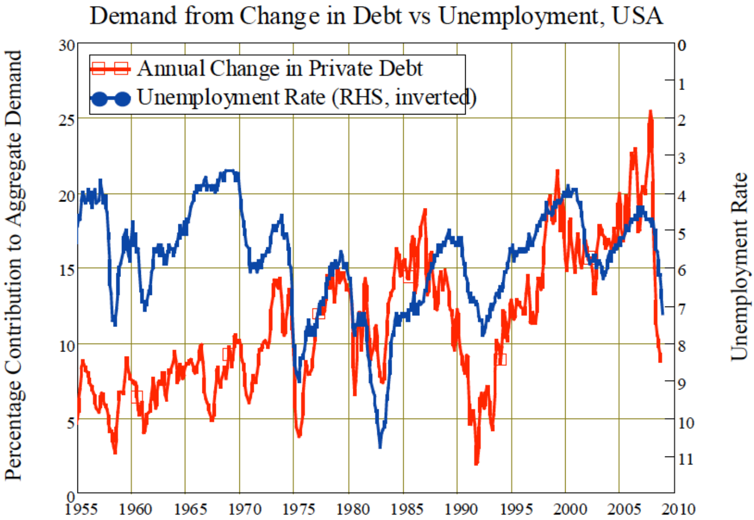


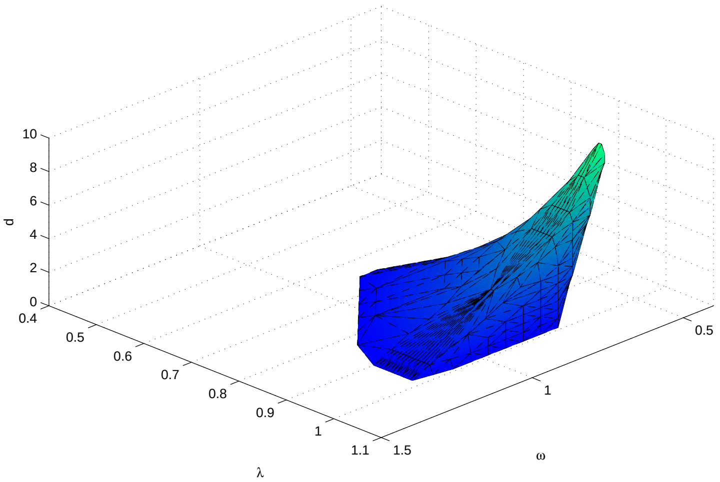
Data detour: debt



Data detour: debt and employment



Basin of convergence for Keen model



Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y + P$$
$$\dot{P} = \Psi(g(\omega, d)P$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g = \frac{\kappa(1-\omega-rd)}{\nu} - \delta.$$

Ponzi financing - Differential equations

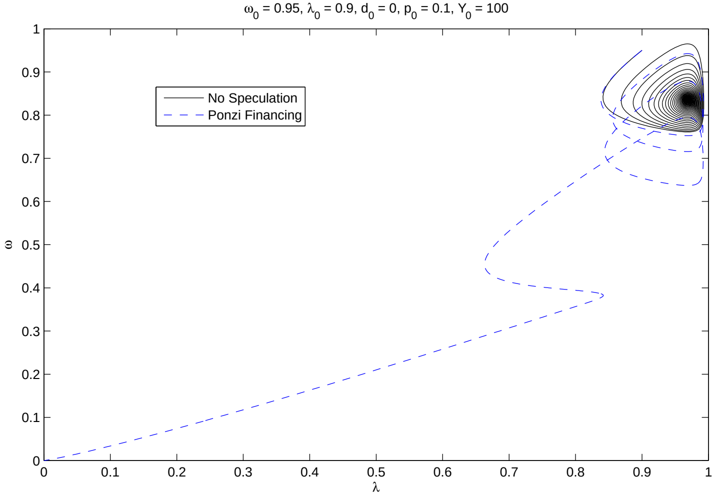
With Ponzi financing the dynamical system becomes

$$
\begin{aligned}
\dot{\omega} &= \omega \left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda \left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d \left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega) + p \\
\dot{p} &= p \left[\Psi\left(\frac{\kappa(1-\omega-rd)}{\nu} - \delta\right) - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right]
\end{aligned}
\tag{4}
$$

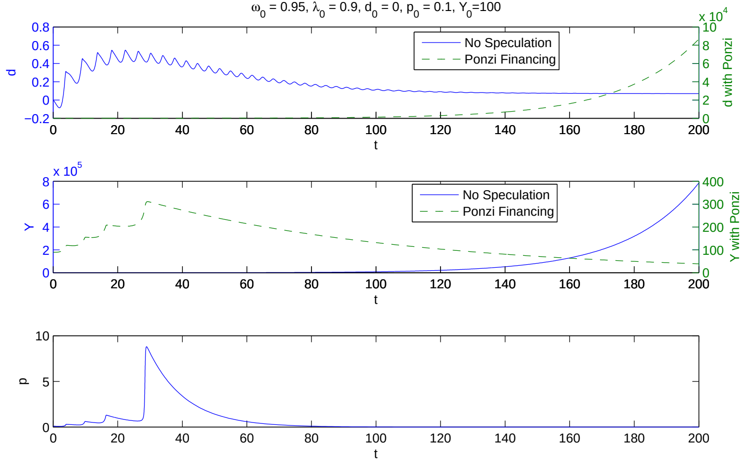
Ponzi financing - Equilibria and stability

- We find that $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1, 0)$ is a stable equilibrium iff
$$\Psi(\alpha+\beta) < \alpha+\beta.$$
- Introducing $u = 1/d$ we find that
$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2, \overline{p}) = (0, 0, +\infty, 0)$$
is stable iff
$$\Psi(g_0) < g_0.$$
- Moreover, introducing , $x = 1/p$ and $v = p/d$ we find that
$$(\overline{\omega}_3, \overline{\lambda}_3, \overline{d}_3, \overline{p}) = (0, 0, +\infty, +\infty)$$
is stable iff
$$g_0 < \Psi(g_0) < r.$$

Example 4: effect of Ponzi financing



Example 4 (continued): effect of Ponzi financing



Introducing a government sector

- A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

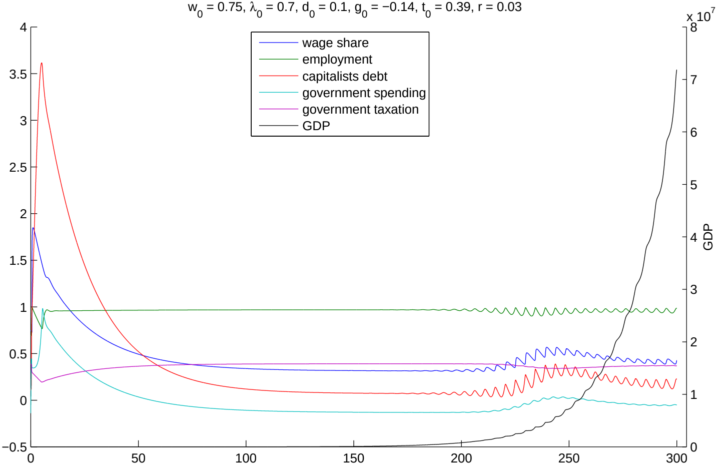
$$\dot{G} = \Gamma(\lambda)Y$$
$$\dot{T} = \Theta(\pi)Y$$

- Defining $g = G/Y$ and $t = T/Y$, the net profit share is now

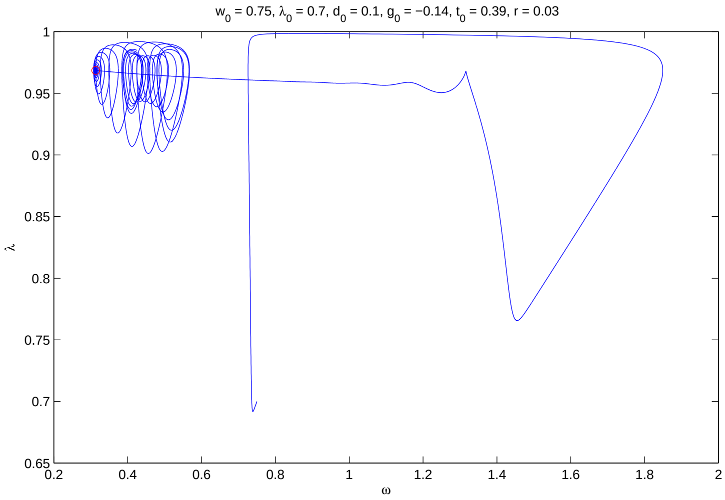
$$\pi = 1 - \omega - rd + g - t$$

- The new 5-dimensional system displays more local fluctuations, but no breakdown for the same initial conditions as before.

Example 5: stabilizing government



Example 5 (continued): stabilizing government



Next steps

- Model prices for capital goods P_k and commodities P_c explicitly (Kaleckian mark-up theory, inflation, etc)
- Introduce noise (stochastic interest rates, risk premium, etc)
- Link with an explicit stochastic model for asset price S_t subject to crashes
- Calibrate to macroeconomic time series.

Concluding thoughts

- Solow (1990): The true test of a simple model is whether it helps us to make sense of the world. Marx was, of course, dead wrong about this. We have changed the world in all sorts of ways, with mixed results; the point is to interpret it.
- Schumpeter (1939): Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them.