Bringing Tobin back: asset price dynamics and portfolio selection in macroeconomics

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James Tobin's contributions to economics

- Tobin received the 1981 Nobel Memorial Prize "for his analysis of financial markets and their relations to expenditure decisions, employment, production and prices".
- Well-known contributions included: foundations of modern portfolio theory (with Markowitz), in particular the Separation Theorem (1958), life-cycle model of consumption, Tobit estimator, Tobin's q, Tobin's tax, ...
- Key forgotten contribution: financial intermediation, portfolio balances, flow of funds models and the credit channel.

Tobin 1969: A General Equilibrium Approach to Monetary Theory

- Specification of (i) a menu of assets, (ii) the factors that determine the demands and supplies of the various assets, and (iii) the manner in which asset prices and interest rates clear these interrelated markets.
- Spending decisions are (provisionally) independent from portfolio decisions.
- Each asset i has a rate of return r_i and each sector j has a net demand f_{ij} for asset i.
- Adding up constraint: for each rate of return r_k,

$$\sum_{i=1}^{n} \frac{\partial f_{ij}}{\partial r_k} = 0.$$

- Paper proceeds to analyze several special cases: money-capital, money-treasuries-capital, bank deposits and loans.

Stock-Flow Consistent models

- Stock-flow consistent models emerged in the last decade as a common language for many heterodox schools of thought in economics.
- They consider both real and monetary factors simultaneously.
- Specify the balance sheet and transactions between sectors.
- Accommodate a number of behavioural assumptions in a way that is consistent with the underlying accounting structure.
- Reject silly (and mathematically unsound!) hypotheses such as the RARE individual (representative agent with rational expectations).
- See Godley and Lavoie (2007) for the full framework.

Balance Sheets

Balance Sheet	Households	Firms		Banks	Central Bank	Government	Sum
		current	capital				
Cash	$+H_h$			$+H_b$	$-H$		0
Deposits	$+M_h$		$+M_f$	$-M$			0
Loans			$-L$	$+L$			0
Bills	$+B_h$			$+B_b$	$+B_c$	$-B$	0
Equities	$+p_f E_f + p_b E_b$		$-p_f E_f$	$-p_b E_b$			0
Advances				$-A$	$+A$		0
Capital			$+pK$				pK
Sum (net worth)	V_h	0	V_f	V_b	0	$-B$	pK

Table: Balance sheet in an example of a general SFC model.

Transactions

Transactions	Households	Firms current	Firms capital	Banks	Central Bank	Government	Sum
Consumption	$-pC_h$	$+pC$		$-pC_b$			0
Investment		$+pI$	$-pI$				0
Gov spending		$+pG$				$-pG$	0
Acct memo [GDP]		$[pY]$					
Wages	$+W$	$-W$					0
Taxes	$-T_h$	$-T_f$				$+T$	0
Interest on deposits	$+r_M.M_h$	$+r_M.M_f$		$-r_M.M$			0
Interest on loans		$-r_L.L$		$+r_L.L$			0
Interest on bills	$+r_B.B_h$			$+r_B.B_b$	$+r_B.B_c$	$-r_B.B$	0
Profits	$+\Pi_d + \Pi_b$	$-\Pi$	$+\Pi_u$	$-\Pi_b$	$-\Pi_c$	$+\Pi_c$	0
Sum	S_h	0	$S_f - pI$	S_b	0	S_g	0

Table: Transactions in an example of a general SFC model.

Flow of Funds

Flow of Funds	Households	Firms		Banks	Central Bank	Government	Sum
		current	capital				
Cash	$+\dot{H}_h$			$+\dot{H}_b$	$-\dot{H}$		0
Deposits	$+\dot{M}_h$		$+\dot{M}_f$	$-\dot{M}$			0
Loans			$-\dot{L}$	$+\dot{L}$			0
Bills	$+\dot{B}_h$			$+\dot{B}_b$	$+\dot{B}_c$	$-\dot{B}$	0
Equities	$+p_f\dot{E}_f + p_b\dot{E}_b$		$-p_f\dot{E}_f$	$-p_b\dot{E}_b$			0
Advances				$-\dot{A}$	$+\dot{A}$		0
Capital			$+pI$				pI
Sum	S_h	0	S_f	S_b	0	S_g	pI
Change in Net Worth	$(S_h + \dot{p}_f E_f + \dot{p}_b E_b)$		$(S_f - \dot{p}_f E_f + \dot{p}K - p\delta K)$	$(S_b - \dot{p}_b E_b)$		S_g	$\dot{p}K + p\dot{K}$

Table: Flow of funds in an example of a general SFC model.

Goodwin Model - SFC matrix

Balance Sheet	Households	Firms		Sum
		current	capital	
Capital			$+pK$	pK
Sum (net worth)	0	0	V_f	pK
Transactions				
Consumption	$-pC$	$+pC$		0
Investment		$+pI$	$-pI$	0
Acct memo [GDP]		$[pY]$		
Wages	$+W$	$-W$		0
Profits		$-\Pi$	$+\Pi_u$	0
Sum	0	0	0	0
Flow of Funds				
Capital			$+pI$	pI
Sum	0	0	Π_u	pI
Change in Net Worth	0	$pI + \dot{p}K - p\delta K$		$\dot{p}K + p\dot{K}$

Table: SFC table for the Goodwin model.

Goodwin Model - Differential equations

- Define

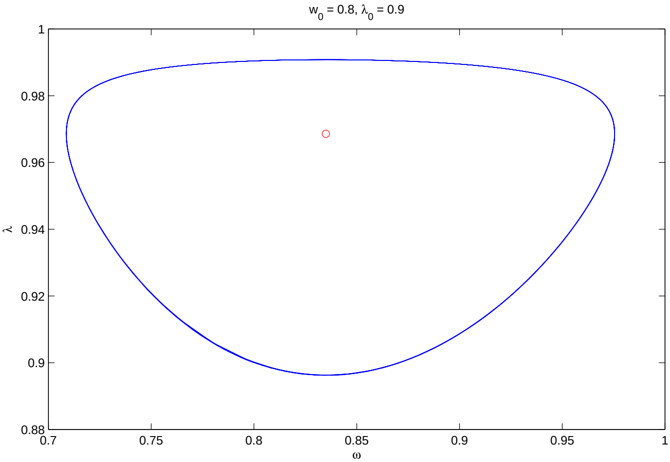
$$\omega = \frac{\mathrm{w}\ell}{pY} = \frac{\mathrm{w}}{pa} \quad \text{(wage share)}$$
$$\lambda = \frac{\ell}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

- It then follows that

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} - \frac{\dot{p}}{p} - \frac{\dot{a}}{a} = \Phi(\lambda, i, i^e) - i - \alpha$$
$$\frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\nu} - \alpha - \beta - \delta$$

- In the original model, all quantities were real (i.e divided by p), which is equivalent to setting $i = i^e = 0$.

Example 1: Goodwin model



Testing Goodwin on OECD countries

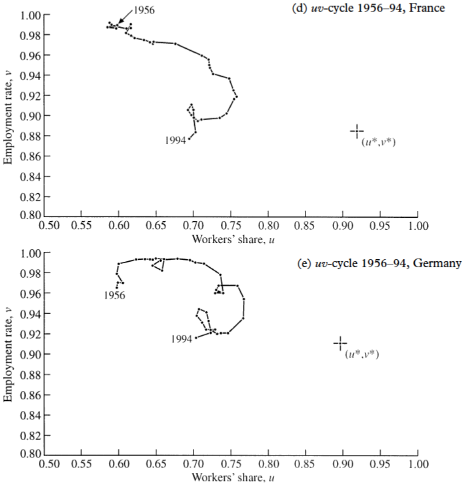


Figure: Harvie (2000)

Correcting Harvie

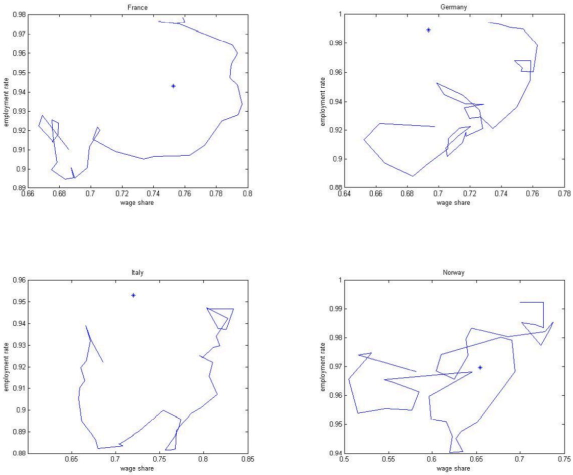


Figure: Grasselli and Maheshwari (2014, in progress)

SFC table for Keen (1995) model

Balance Sheet	Households	Firms current	Firms capital	Banks	Sum
Deposits	$+D$			$-D$	0
Loans			$-L$	$+L$	0
Capital			$+pK$		pK
Sum (net worth)	V_h	0	V_f	0	pK
Transactions					
Consumption	$-pC$	$+pC$			0
Investment		$+pI$	$-pI$		0
Acct memo [GDP]		$[pY]$			
Wages	$+W$	$-W$			0
Interest on deposits	$+rD$			$-rD$	0
Interest on loans		$-rL$		$+rL$	0
Profits		$-\Pi$	$+\Pi_u$		0
Sum	S_h	0	$S_f - pI$	0	0
Flow of Funds					
Deposits	$+\dot{D}$			$-\dot{D}$	0
Loans			$-\dot{L}$	$+\dot{L}$	0
Capital			$+pI$		pI
Sum	S_h	0	Π_u	0	pI
Change in Net Worth	S_h	$(S_f + \dot{p}K - p\delta K)$			$\dot{p}K + p\dot{K}$

Table: SFC table for the Keen model.

Keen model - Investment function

- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

where $\kappa(\cdot)$ is a nonlinear increasing function of profits $\pi = 1 - \omega - rd$.

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y$$

Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned} \tag{1}$$

Example 2: convergence to the good equilibrium in a Keen model

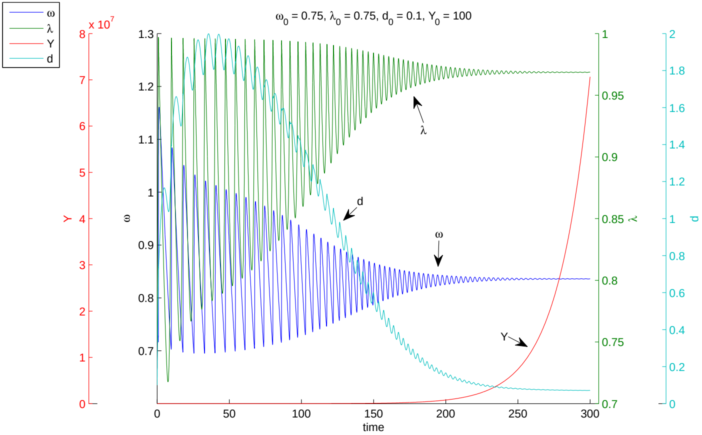


Figure: Grasselli and Costa Lima (2012)

Example 3: explosive debt in a Keen model

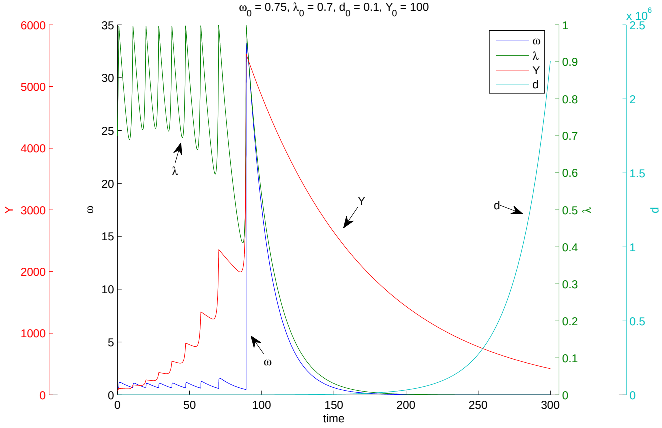


Figure: Grasselli and Costa Lima (2012)

Basin of convergence for Keen model

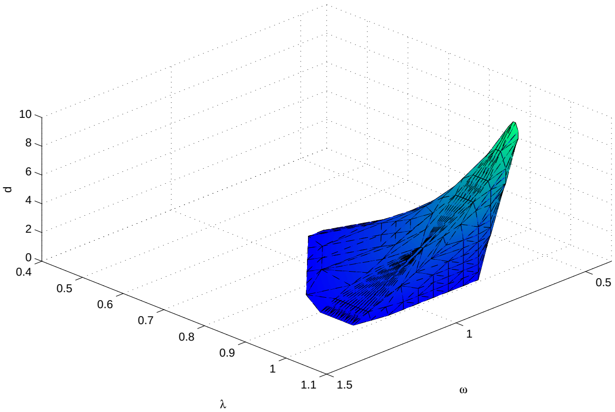


Figure: Grasselli and Costa Lima (2012)

Extensions

- Costa Lima and Nguyen (2014) add random productivity to the Goodwin model and prove the existence of stochastic orbits for the generalized Lotka-Volterra system.

- Costa Lima, Grasselli, Wang and Wu (2014) show that government spending and taxation can prevent the bad equilibrium with infinite debt and zero employment.

- Choi and Grasselli (2014, in progress) characterize the Great Moderation in the U.S. as a Shilnikov bifurcation for the Keen model.

Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y + P$$
$$\dot{P} = \Psi(g(\omega,d)P$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g = \frac{\kappa(1-\omega-rd)}{\nu} - \delta.$$

Example 4: effect of Ponzi financing

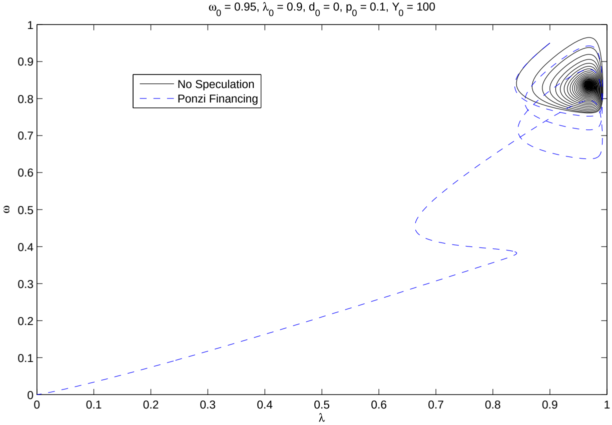


Figure: Grasselli and Costa Lima (2012)

Stock prices

- Consider a stock price process of the form

$$\frac{dS_t}{S_t} = r_b dt + \sigma dW_t + \gamma \mu_t dt - \gamma dN^{(\mu_t)}$$

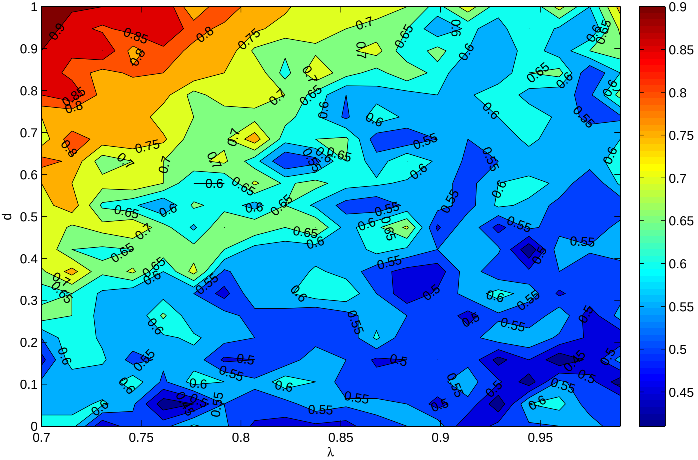
where N_t is a Cox process with stochastic intensity $\mu_t = M(p(t))$.

- The interest rate for private debt is modelled as $r_t = r_b + r_p(t)$ where

$$r_p(t) = \rho_1 (S_t + \rho_2)^{\rho_3}$$

Stability map

Stability map for ω_0 = 0.8, p_0 = 0.01, S_0 = 100, T = 500, dt = 0.005, # of simulations = 100



The Great Moderation in the U.S. - 1984 to 2007

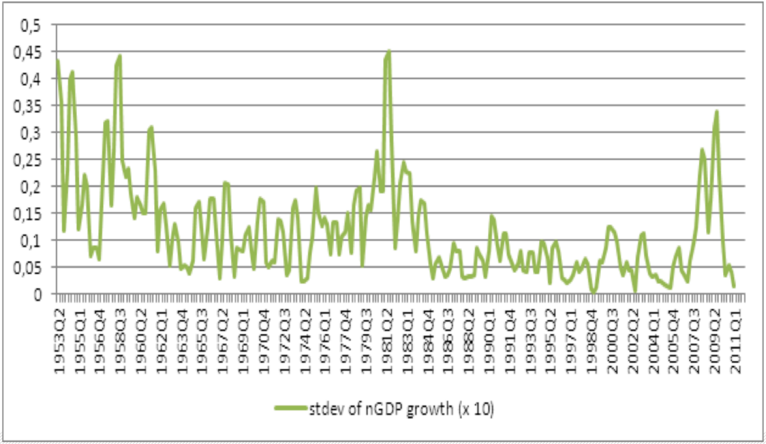


Figure: Grydaki and Bezemer (2013)

Possible explanations

- Real-sector causes: inventory management, labour market changes, responses to oil shocks, external balances , etc.
- Financial-sector causes: credit accelerator models, financial innovation, deregulation, better monetary policy, etc.
- Grydaki and Bezemer (2013): growth of debt in the real sector.

Bank credit-to-GDP ratio in the U.S

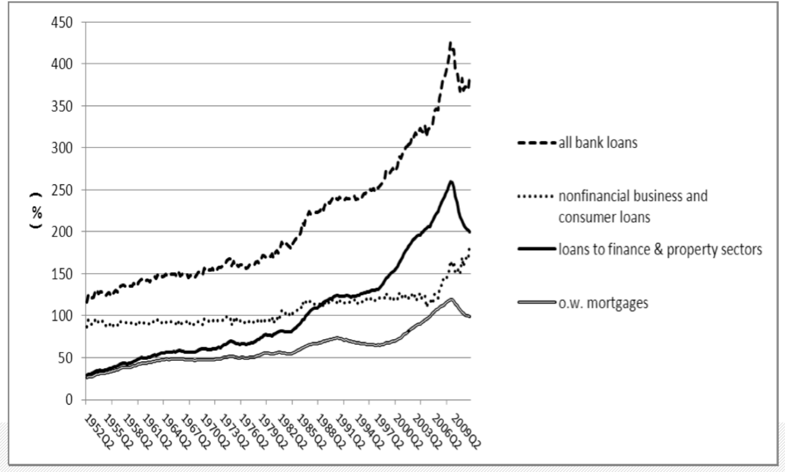


Figure: Grydaki and Bezemer (2013)

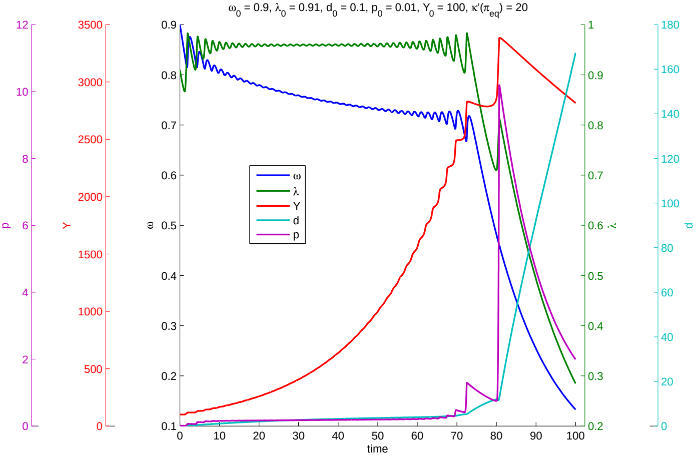
Excess credit growth moderated output volatility during, but not before the Great Moderation

Before the Great Moderation	*During the Great Moderation*
change in interest rate (-) => output volatility	excess credit growth (-) => output volatility
change in interest rate (+) => inflation	output volatility (+) => excess credit growth
excess credit growth (+) => change in interest rate	output volatility (-) => change in interest rate
	excess credit growth (+) => change in interest rate
	inflation (+) => change in interest rate

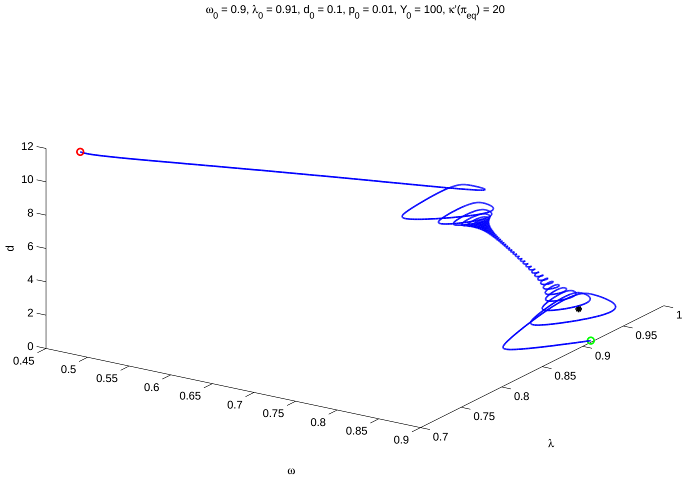
Note: In the table, *x* (-) => *y* denotes that a one-standard deviation shock in variable *x* impacts negatively on the change of variable *y*. Similarly, *x* (+) => *y* indicates a positive impact.

Figure: Grydaki and Bezemer (2013)

Example 5: strongly moderated oscillations



Example 5 (cont): Shilnikov bifurcation



Shortcomings of Goodwin and Keen models

- No independent specification of consumption (and therefore savings) for households:

$$C = W, \quad S_h = 0 \qquad \text{(Goodwin)}$$

$$C = (1-\kappa(\pi))Y, \quad S_h = \dot{D} = \Pi_u - I \qquad \text{(Keen)}$$

- Full capacity utilization.
- Everything that is produced is sold.
- No active market for equities.
- Skott (1989) uses prices as an accommodating variable in the short run.
- Chiarella, Flaschel and Franke (2005) propose a dynamics for inventory and expected sales.
- Grasselli and Nguyen (2014) provide a synthesis, including equities and Tobin's portfolio choices.

Inventory dynamics

- Denoting demand by Y_d, expected sales by Y_e and capacity utilization by $u = Y/Y_{\max}$ we obtain the system

$$\begin{aligned}
\dot{\omega}_e &= \omega_e \left[\Phi(\lambda) - \alpha + (1-\eta_p)\gamma(1-m\omega_e)\right] \\
\dot{\lambda} &= \lambda \left[g_e y_e + g_d y_d - \eta_v - \alpha - \beta\right] \\
\dot{d}_e &= d_e \left[-g_e y_e - g_d y_d + \eta_v + \gamma(1-m\omega_e) + r\right] \\
&\quad + \left[\frac{\kappa(\pi_e) + \eta_u(u-\overline{u})}{u} - (1-\omega_e)y_d\right] \qquad , \\
\dot{y}_e &= y_e(\alpha + \beta - \eta_d - g_e y_e - g_d y_d + \eta_v) + \eta_d y_d \\
\dot{u} &= u \left[g_e y_e + g_d y_d - \eta_v - \frac{\kappa(\pi_e) + \eta_u(u-\overline{u})}{\nu} + \delta\right]
\end{aligned}$$

of which the previous model is a special case.

Firm decisions

- Suppose now that firms finance new investment by issuing equities E at price p_e as well as new loans.
- Assuming that undistributed profits take the form $s_f\Pi$ for a constant s_f, the amount needed to be raised externally for new investment is $pI_k - s_f\Pi$, according to the proportions

$$\dot{D} = \nu_D[pI_k - s_f\Pi]$$
$$p_e\dot{E} = \nu_E[pI_k - s_f\Pi],$$

with $\nu_D + \nu_E = 1$.

- Here both I_k and ν_E can be functions of Tobin's $q = \frac{p_e E}{pK}$.

Household decisions

- On the other hand, the budget constraint for households is

$$W + (1 - s_f)\Pi + rD = pC + \dot{D} + p_e\dot{E},$$

whereas their portfolio allocation is

$$\begin{aligned} p_e E &= f_e(r_e^e)X_h \\ D &= 1 - f_e(r_e^e)X_h, \end{aligned}$$

where

$$\begin{aligned} r_e^e &= \frac{(1 - s_f)\Pi}{p_e E} + \pi_e^e \\ \dot{\pi}_e^e &= \beta_{\pi_e}\left(\frac{\dot{p}_e}{p_e} - \pi_e^e\right) \end{aligned}$$

- This leads to an extended system with two more equations for $\dot{e}/e$ and $\dot{\pi}_e^e$.

Concluding remarks

- Macroeconomics is too important to be left to macroeconomists.
- Since Keynes's death it has developed in two radically different approaches:
  1. The dominant one has the appearance of mathematical rigour (the SMD theorems notwithstanding), but is based on implausible assumptions, has poor fit to data in general, and is disastrously wrong during crises. Finance plays a negligible role
  2. The heterodox approach is grounded in history and institutional understanding, takes empirical work much more seriously, but is generally averse to mathematics. Finance plays a major role.
- It's clear which approach should be embraced by mathematical finance.

Thank you!


FINAL
30th June 2002
2-0
Brazil
Germany
sporting spirit and fellowship of all the players, this monument is dedicated to the participants
football fans who gave their unstinted support.