An agent-based model for bank formation, bank runs and interbank networks

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CMS Winter Meeting, December 11, 2011

The quest to understand banking crises

- Financial crises in the past 800 years encompass:
  1. sovereign defaults
  2. currency debasement and inflation
  3. exchange rate crises
  4. banking crises
- Graduating from banking crises has eluded developed and developing countries alike - Reinhart and Rogoff (2009).
- Individual banks are subject to runs, largely addressed by deposit insurance, capital requirements, and regulation.
- However, the principles that govern individual prudence do not necessarily apply to systems as a whole.
- Financial innovation and integration leads to highly interconnected, complex and potentially fragile banking systems.
- Systemic crises are essentially stories of contagion, interdependence, interaction and trust - Kirman (2010).

Agent-Based Models in Economics

- Modern macroeconomic theory (e.g 'sophisticated' DSGE models) is hopeless inadequate to deal with banking crises.
- Representative agents, neutrality of money, stationarity of expectations, and assumed equilibrium states are non-starters for the problem at hand.
- Agent-based computational economics (ACE) has emerged as an alternative.
- Agents have rational objectives, but realistic computational devices (inductive learning, bounded memory, limited information, war games, etc).
- Interactions are modelled directly, without fictitious clearing mechanisms.
- Hierarchical structures (i.e, banks are agents, but so are their clients, as well as the government).
- Equilibrium is just one possible outcome, not assumed a priori.

Liquidity preferences

- An asset is illiquid if its liquidation value at an earlier time is less than the present value of its future payoff.
- For example, an asset can pay $1 \leq r_1 \leq r_2$ at dates $T = 0, 1, 2$.
- The lower the ratio r_1/r_2 the less liquid is the asset.
- At time $t = 0$, consumers don't know in which future date they will consume.
- The consumer's expected utility is

$$wU(r_1) + (1 - w)U(r_2),$$

where w is the proportion of early consumers (type 1).

- Sufficiently risk-averse consumers prefer the liquid asset.

Example: Diamond (2007)

- Let $A = (r_1 = 1, r_2 = 2)$ represent an illiquid asset and $B = (r_1 = 1.28, r_2 = 1.813)$ a liquid one.
- Assume investors with power utility $u(c) = 1 - c^{-1}$ and $w = 1/4$.
- The expected utility from holding the illiquid asset is

$$E[u(c)] = \frac{1}{4}u(1) + \frac{3}{4}u(2) = 0.375$$

- By comparison, the expected utility from holding the liquid asset is

$$E[u(c)] = \frac{1}{4}u(1.28) + \frac{3}{4}u(1.813) = 0.391$$

- Observe, however, that risk-neutral investors would prefer the illiquid asset, since:

$$E[A] = 1.75 > 1.68 = E[B]$$

Liquidity risk sharing with public information

- Consider an economy with dates $T = 0, 1, 2$ and consumer preferences given by

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with prob } \omega \\ u(c_2) & \text{with prob } 1-\omega \end{cases} \tag{1}$$

- Agents are endowed with one unit of the numeraire at time 0 and must decide either to hold it or to invest in an illiquid asset $(1, R)$.
- Denoting the consumption of agent of type i at time k by c_k^i the optimal risk sharing for *publicly* observed preferences is

$$c_1^2 = c_2^1 = 0 \tag{2}$$

$$u'(c_1^1) = Ru'(c_2^2) \tag{3}$$

$$\omega c_1^1 + (1-\omega)\frac{c_2^2}{R} = 1 \tag{4}$$

A model for banks - Diamond and Dybvig (1983)

- However, liquidity preferences are private unverifiable information !
- Fortunately, the optimal solution satisfies the self–selection condition $1 < c_1^1 < c_2^2 < R$, which in turn implies that there is a contract that implements it as a Nash equilibrium.
- Suppose now that a bank offers a fixed claim r_1 per unit deposited at time 0.
- Assume that withdrawers are served sequentially in random order until bank runs out of assets.
- Denoting by f the total fraction of withdrawers, we see that $r_1 = c_1^1$ and $f = \omega$ is such equilibrium.
- However, it is clear that $f = 1$ (run) is also an equilibrium.

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Society

- Consider N heterogeneous agents with liquidity preferences at times t_k given by independent uniform random variables ω^i on $[0,1]$: if $\omega^i < p$, agent i is said to be of type 1 (impatient), otherwise it is said to be of type 2 (patient).
- At t_{k+1}, define

$$\widetilde{\omega}_k^i = \omega^i + (-1)^{b_k^i} \frac{\varepsilon_k^i}{2}, \tag{5}$$

where $b_k^i \in \{0,1\}$ are Bernoulli random variables and ε_k^i are uniformly distributed on $[0,1]$. Setting $q = 2p - 1/2$, agent i is then deemed to be impatient if $\widetilde{\omega}^i < q$ and patient otherwise.

- Because of anticipated shocks, individuals explore the society searching to partners to exchange investments.

Searching for partners

- We impose some constrains on the individual capacity to go around and seek other individuals to trade.
- This reflects the inherited limited capability of information gathering and environment knowledge of individual agents.
- We use a combination of Von Neumann and Moore neighborhood:

5	1	6
2	X	3
7	4	8

Matching example

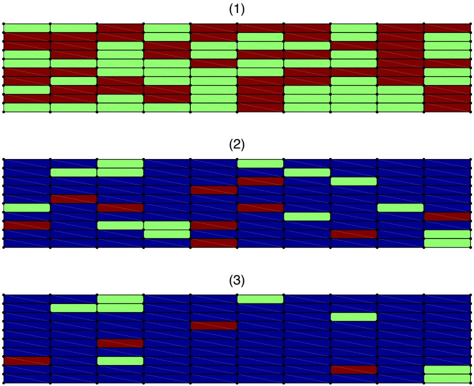


Figure: Society, preference shock, and search for partners.

Inductive reasoning

- We follow the inductive reasoning proposed by Arthur (2000) for individuals with bounded rationality dealing with complex environments.
- We assume agents make predictions using a memory of 5 periods.
- All agents have a set of 7 predictors as follows:
  1. Today would be the same as last period.
  2. Today would be the same as two periods ago.
  3. Today would be the same as three periods ago.
  4. Today would be the same as four periods ago.
  5. Today would be the same as five periods ago.
  6. Today would be the same as the mode for the last three periods.
  7. Today would be the same as the mode for the last five periods.

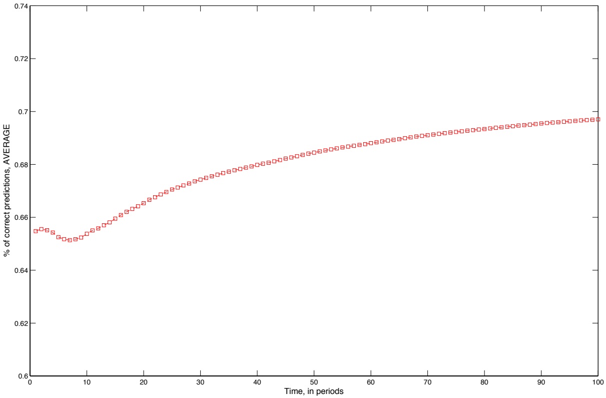
Learning and Predicting

- Each predictor makes one of the following forecasts:
  1. N = agent will not need a partner
  2. G = agent will need a partner and will find one
  3. B = agent will need a partner and will not find one
- Depending on the realized outcome, a predictor's strength gets updated by

$$\Delta S = \begin{cases} +1 & \text{if the forecast is correct} \\ -1 & \text{if the forecast is incorrect} \end{cases}$$

Learning simulation

We use 400 persons over a time span of 100 periods in a simulation with 100 realizations:



To join or not to join a bank

- Suppose that agents need to decide between the liquid asset $(1, 1)$, the illiquid asset $(r < 1, R > 1)$ or joining the bank and receiving $(c_1 > 1, c_2 < R)$.
- For example, an agent who current has late preferences might have the following payoff table:

	forecast	strength	payoff (join)	payoff (not join)
1	N	-2	c_2	R
2	G	0	c_1	1
3	N	$+1$	c_2	R
4	B	-1	c_1	r
5	G	$+1$	c_1	1
6	N	0	c_2	R
7	B	$+2$	c_1	r

- The decision is based on the weighted sum of payoffs.

Bank birth

- We follow the work of Howitt and Clower (1999, 2007) on the emergence of economic organizations.
- A randomly selected agent i is hit by the 'idea of entrepreneurship' and makes an initial estimate $W_k^i \in \{0, 1/9, 2/9, \ldots, 1\}$ of the fraction of early consumers amongst its neighbours.
- The bank is establish if there are x_k^i and y_k^i such that $x_k^i + y_k^i \leq 1$ and

$$y_k^i = c_1 W_k^i$$
$$R x_k^i = c_2(1 - W_k^i),$$

  where (c_1, c_2) is the promised consumption.
- Individuals become aware of bank existence only if the bank lies in their neighbourhood
- In addition we give the bank the reach of its new members

Experiment: bank formation

Experiment (continued): established banks

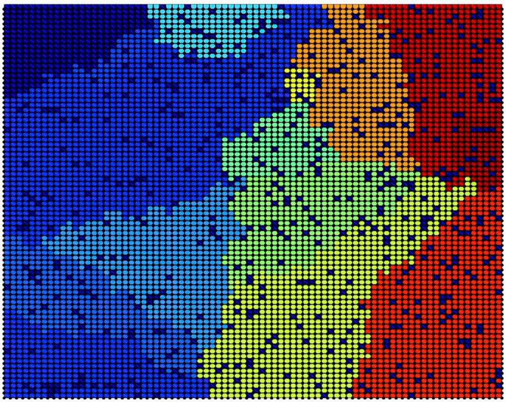
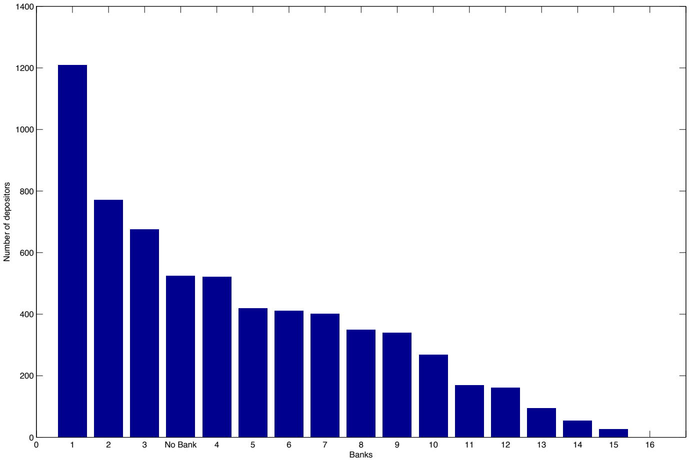


Figure: Banks at T=100 with $c_1 = 1.1$, $c_2 = 1.5$ and $R = 2$

Experiment (continued): number of depositors



Dynamic allocation

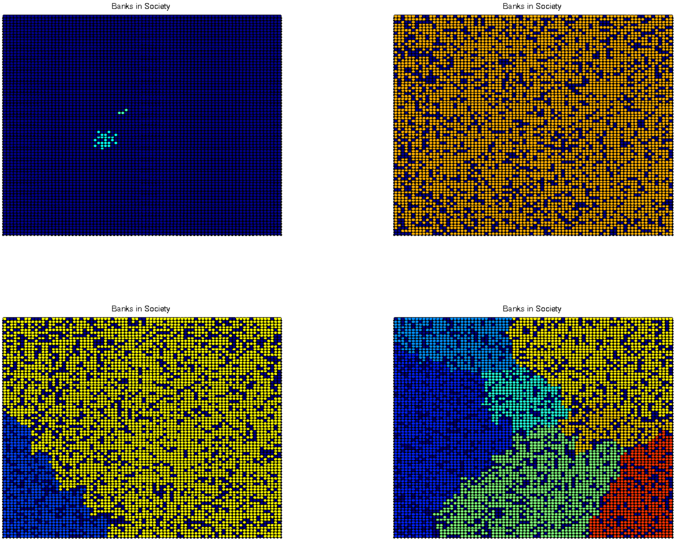
- In the previous section we assumed that an agent never leaves a bank after joining.
- To model bank failures and runs we need a learning mechanism for banks themselves.
- Having made the allocation (x^i_k, y^i_k) based on W^i_k, banks fail or survive according to the realized $\overline{W}^i_k$.
- We say that a bank is subject to a run if late consumers receive less than c_1 at the end of the period.
- If a bank survives at period k, it updates the estimate of early consumers according to

$$W^i_{k+1} = W^i_k + \alpha(\overline{W}^i_k - W^i_k) \tag{6}$$

reflecting adaptation through a parameter $\alpha \in (0, 1)$.

Experiment: bank formation and runs

Experiment: established banks (with possible runs)



Banks and learning

- As before, banks update their estimate of the fraction of early consumers according to (6).
- In addition, they deem the estimate to be adequate if the fraction of reserves lost in a given period is less than a certain threshold.
- They use the same set of predictors as clients to forecast the adequacy of their estimates as being ‘adequate’, ‘inadequate’ or ‘undetermined’.
- Banks with inadequate or undetermined estimates have an incentive to exchange deposits with other banks and try to protect their reserves.

Experiment: adequacy of estimates through time

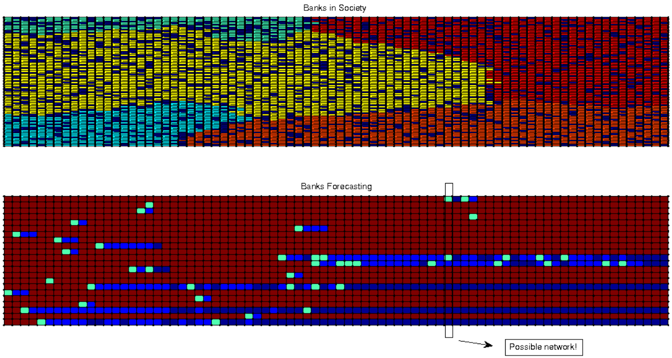


Figure: Banks at T=100 with $c_1 = 1.1$, $c_2 = 1.5$ and $R = 2$ and adequacy of estimates over time.

Experiment: possible network

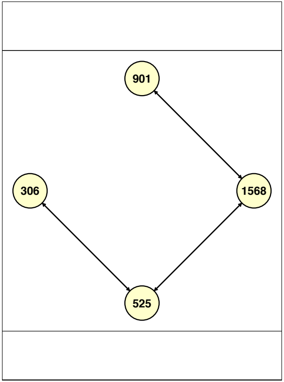
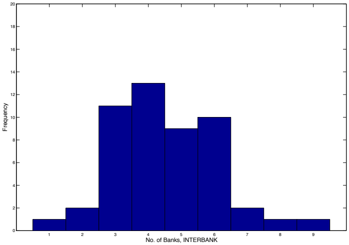
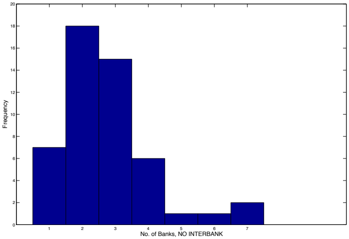


Figure: Snapshot of possible interbank loans

Number of established banks with and without interbank links

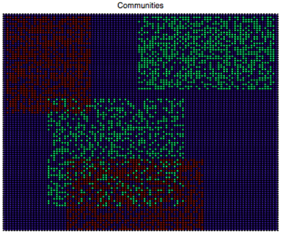


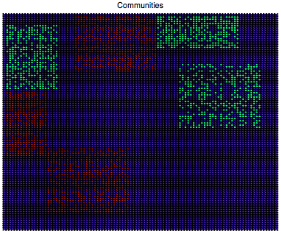


Correlated liquidity shocks

- As in Allen and Gale (2000), we consider regional liquidity shocks in a society with no overall shortage of liquidity.
- We form $2C$ different regions (communities) as follows:
  1. Select $2C$ cells at random to be the base
  2. Choose the largest reach M around the base
  3. Randomly select $2M^2$ cells around the base to form a community
  4. Alter half of the communities to early preferences and half of the communities to late preferences.

Examples of correlated liquidity shocks





Experiment: bank formation and runs with correlated shocks

Experiment: adequacy of estimates through time (with correlated shocks)

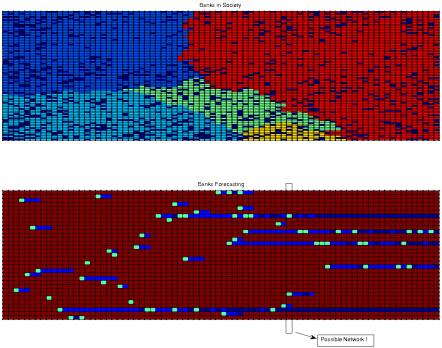
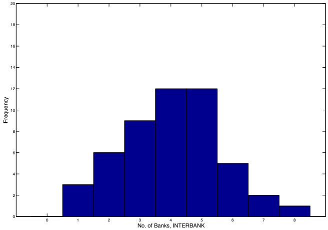
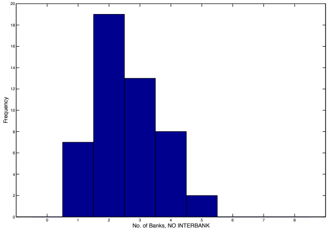


Figure: Banks at T=100 with $c_1 = 1.1$, $c_2 = 1.5$ and $R = 2$ and adequacy of estimates over time.

Number of established banks under correlated shocks





Concluding remarks

- We modelled individual liquidity preferences in a society.
- Changes in preferences lead agents to search for trading partners.
- Banks arise as providers of liquidity, but are inevitably subject to possible runs.
- Interbank loans redistributed the effect of correlated liquidity shocks across the society.
- Ultimately want to adjust model parameters to reproduced different observed networks and use it as a testbed for policy implications.