A dynamical systems model for credit expansion, asset price bubbles and financial fragility

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Outline

Dynamic General Equilibrium views

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- All variables are assumed to be simultaneously in equilibrium.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.
- Money is neutral in its effect on real variables.
- Largely ignores uncertainty by simply subtracting risk premia from all risky returns and treat them as risk-free.

Voices of discontent

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- K. Arrow (1986): "In the aggregate, the hypothesis of rational behavior has in general no implications."
- R. Solow (2006): "Maybe there is in human nature a deep-seated perverse pleasure in adopting and defending a wholly counterintuitive doctrine that leaves the uninitiated peasant wondering what planet he or she is on."

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- Uncertainty in valuation of cash flows (assets) and credit risk (liabilities) drive fluctuations in real demand and investment.
- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.

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- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
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- "Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing".

Goodwin Model (1967) - Assumptions

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Introduction

Goodwin model

Keen model

Ponzi financing

Stabilizing government

- Assume that

$$N(t) = N_0 e^{\beta t} \quad \text{(total labour force)}$$

$$a(t) = a_0 e^{\alpha t} \quad \text{(productivity per worker)}$$

$$Y(t) = \nu K(t) = a(t)L(t) \quad \text{(total yearly output)}$$

where K is the total stock of capital and L is the employed population.

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- Assume further that

$$\dot{\mathrm{w}} = \Phi(\lambda)\mathrm{w} \quad \text{(Phillips curve)}$$
$$\dot{K} = (Y - \mathrm{w}L) - \delta K \quad \text{(Say's Law)}$$

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}L}{Y} = \frac{\mathrm{w}}{a} \quad \text{(wage share)}$$

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- It then follows that

$$\dot{\omega} = \omega(\Phi(\lambda) - \alpha) \tag{1}$$

$$\dot{\lambda} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \beta - \delta\right) \tag{2}$$

Goodwin Model - Properties

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- Provided

$$\begin{aligned} \frac{1}{\nu} - \alpha - \beta - \delta &> 0 \\ \alpha + \phi_0 - \rho_1 &> 0 \end{aligned} \tag{3}$$

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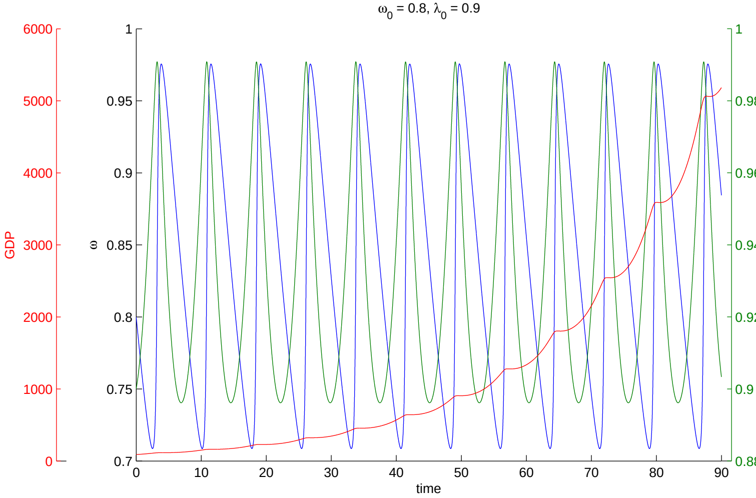
- Moreover the only other equilibrium

$$(\overline{\omega}, \overline{\lambda}) = \left(1 - \nu(\alpha+\beta+\delta), 1 - \sqrt{\frac{\phi_1}{\alpha+\phi_0}}\right) \tag{4}$$

is non-hyperbolic.

Example 1: Goodwin model

Example 1 (continued): Goodwin model



Goodwin Model - Extensions, structural instability, and empirical tests

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- Solow 1990: US post-war data shows three sub-cycles with a "bare hint of a single large clockwise sweep" in the (ω, λ) plot.
- Harview 2000: Data from other OECD confirms the same qualitative features and shows unsatisfactory quantitative estimations.

Testing Goodwin on OECD countries

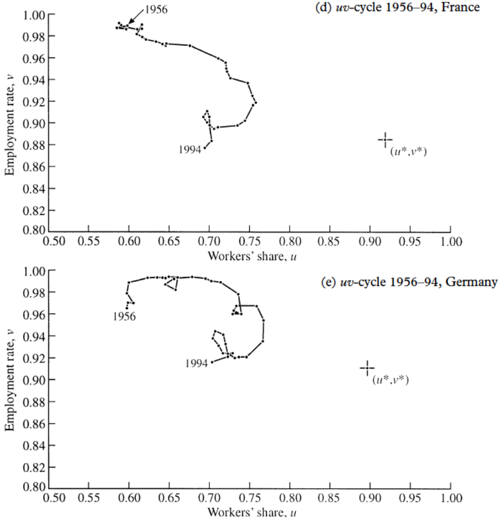

Figure: Source: Harvie (2000)

Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K \qquad (5)$$

where $\kappa(\dot{)}$ is an increasing function the net profit share $\pi = 1 - \omega - rd$.

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$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y$$

- We take

$$\kappa(x) = \kappa_0 + \kappa_1 e^{\kappa_2 x},$$

for constants

$$\kappa_0 < \nu(\alpha+\beta+\delta), \quad \kappa_1 > 0, \quad \kappa_2 > 0. \tag{6}$$

Keen model - Differential Equations

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Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$
\begin{aligned}
\dot{\omega} &= \omega\left[\frac{\phi_1}{(1-\lambda)^2} - (\alpha+\phi_0)\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned}
\tag{7}
$$

Keen model - good equilibrium

- If we define

$$\overline{\pi}_1 = \kappa^{-1}(\nu(\alpha+\beta+\delta)) = \frac{1}{\kappa_2}\log\left(\frac{\nu(\alpha+\beta+\delta)-\kappa_0}{\kappa_1}\right)$$

we see that one possible equilibrium for (7) is

$$\begin{aligned}
\overline{\omega}_1 &= 1 - \overline{\pi}_1 - r\frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_1}{\alpha+\beta} \\
\overline{\lambda}_1 &= 1 - \sqrt{\frac{\phi_1}{\alpha+\phi_0}} \\
\overline{d}_1 &= \frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_1}{\alpha+\beta}
\end{aligned} \tag{8}$$

Keen model - Irrelevant equilibria

- Other equilibrium points are given by

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2) = (0, 0, \overline{d}_2) \tag{9}$$

where $\overline{d}_2$ is any solution of the equation

$$d\left[r - \frac{\kappa(1 - rd)}{\nu} + \delta\right] + \kappa(1 - rd) - 1 = 0$$

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- Another set of equilibrium points are

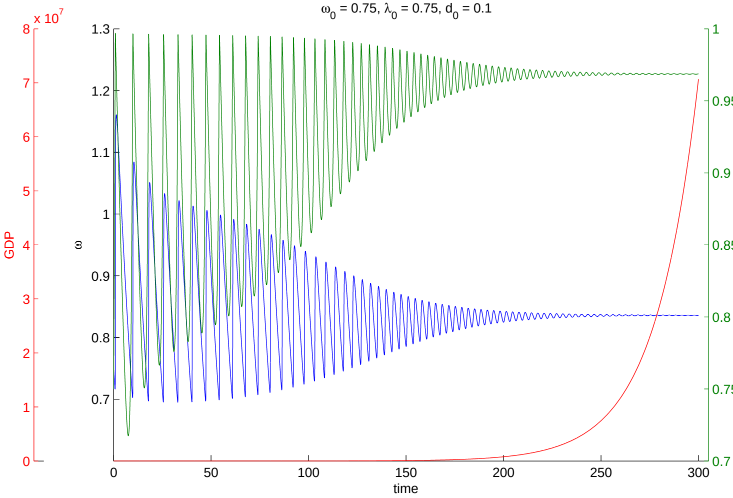
$$(\overline{\omega}_3, \overline{\lambda}_3, \overline{d}_3) = (0, \lambda, \overline{d}_1) \tag{10}$$

provided $1 - r\overline{d}_1 = \overline{\pi}_1$, that is

$$1 - r\frac{\nu(\alpha+\beta+\delta) - \kappa^{-1}(\nu(\alpha+\beta+\delta))}{\alpha+\beta} = \kappa^{-1}(\nu(\alpha+\beta+\delta))$$

Example 2 : convergent Keen model

Example 2 (continued): convergent Keen model



Keen model - Explosive debt

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

$$\dot{\omega} = \omega \left[\frac{\phi_1}{(1-\lambda)^2} - (\alpha + \phi_0) \right]$$

$$\dot{\lambda} = \lambda \left[\frac{\kappa(1-\omega-r/u)}{\nu} - \alpha - \beta - \delta \right] \quad (11)$$

$$\dot{u} = u \left[\frac{\kappa(1-\omega-r/u)}{\nu} - r - \delta \right] - u^2 \left[\kappa(1-\omega-r/u) - (1-\omega) \right]$$

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\end{aligned} \tag{11}$$

- We now see that $(0,0,0)$ is an equilibrium of (11) corresponding to the point

$$(\overline{\omega}_4, \overline{\lambda}_4, \overline{d}_4) = (0, 0, +\infty) \tag{12}$$

for the original system.

Example 3: divergent Keen model

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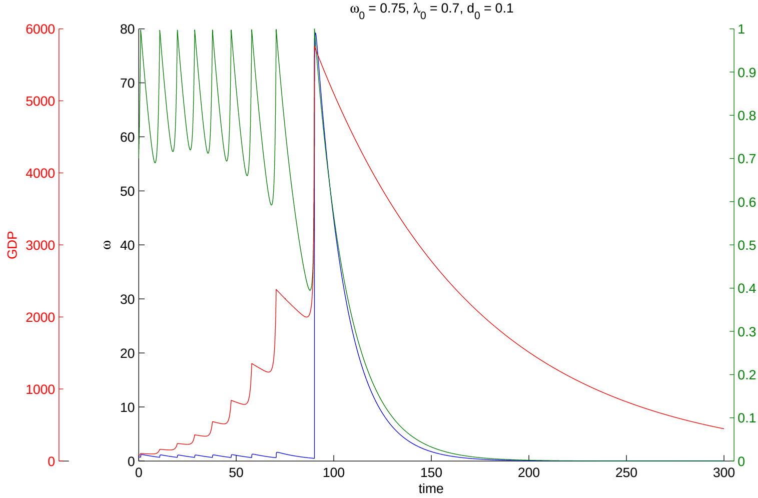
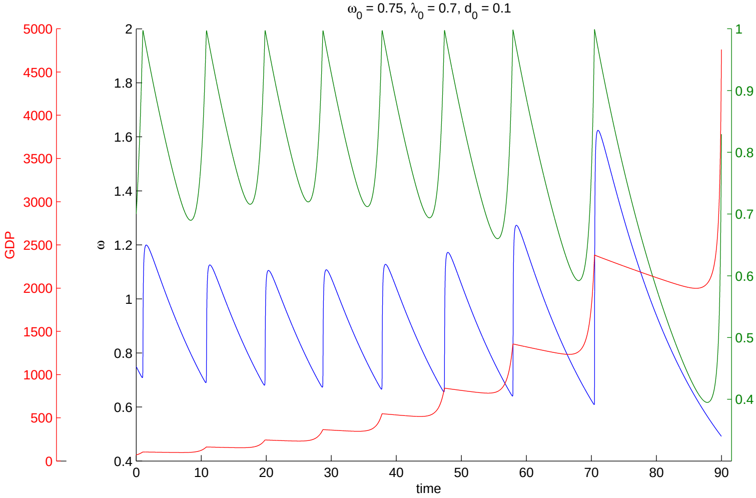
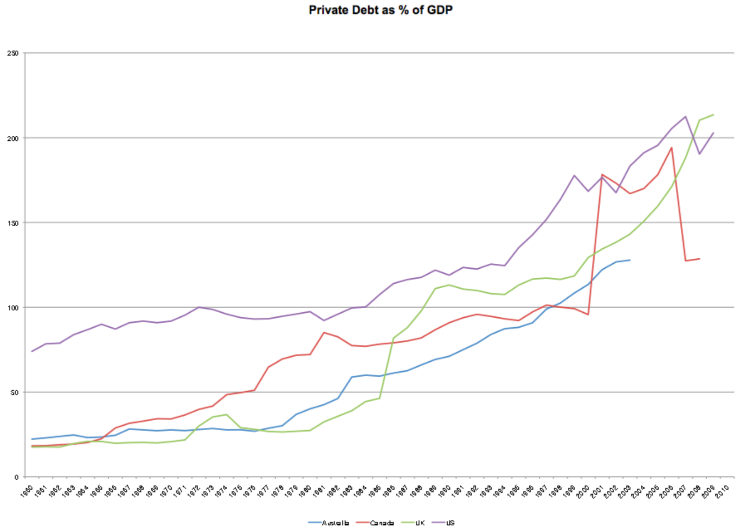


Example 3 (continued): divergent Keen model



Data detour: debt



Data detour: debt and employment

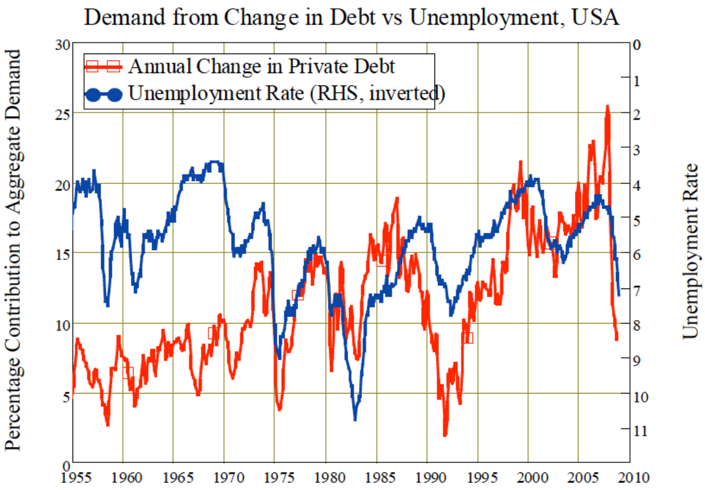


Figure: Source: Keen (2009)

Keen model - Local stability

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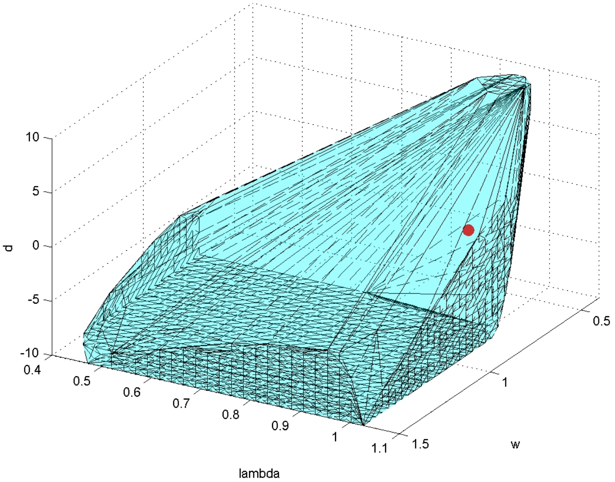
- The point $(0, 0, 0)$ is a stable equilibrium for (11) if and only if

$$\frac{\kappa_0}{\nu} < \min(\alpha + \beta + \delta, r + \delta)$$

Basin of convergence for Keen model



Ponzi financing

To introduce the destabilizing effect of purely speculative investment, Keen (2009) considers a modified version of the previous model with

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y + P$$
$$\dot{P} = \Psi(g)Y$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g = \frac{\kappa(1-\omega-rd)}{\nu} - \delta.$$

Ponzi financing - Differential equations

Denoting $\Psi(\omega, d) = \Psi\left(\frac{\kappa(1-\omega-rd)}{\nu} - \delta\right)$, the system now becomes

$$\dot{\omega} = \omega\left[\frac{\phi_1}{(1-\lambda)^2} - (\alpha + \phi_0)\right]$$

$$\dot{\lambda} = \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right]$$

$$\dot{d} = d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega) + p$$

$$\dot{p} = \Psi(\omega, d) - \left(\frac{\kappa(1-\omega-rd)}{\nu} - \delta\right)p$$

Ponzi financing - Equilibria

- The finite debt equilibrium for the new system has $\overline{\pi}_1$ and $\overline{\lambda}_1$ as before, but

$$\overline{\omega}_1 = 1 - \overline{\pi}_1 - r\overline{d}_1$$

$$\overline{d}_1 = \frac{\nu(\alpha + \beta + \delta) - \overline{\pi}_1 + \overline{p}_1}{\alpha + \beta}$$

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- On the other hand, introducing $u = 1/d$ and $v = 1/p$ we find that

$$(\omega, \lambda, d, p) = (0, 0, +\infty, -\infty)$$

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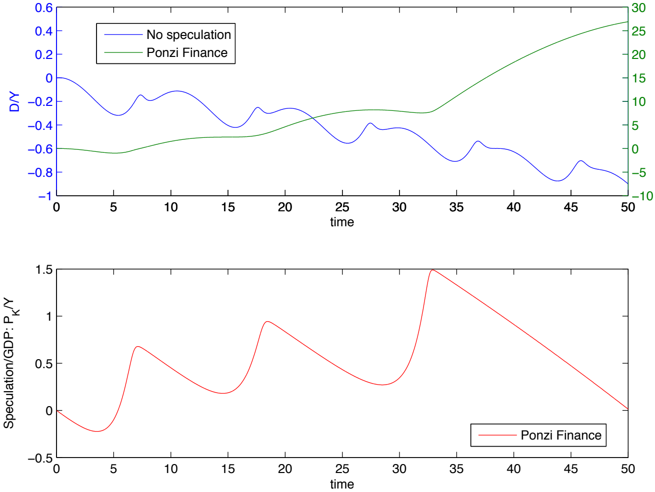
correspond to stable equilibria for the modified system.

Example 4: effect of Ponzi financing



Introducing a government sector

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- Defining $g = G/Y$ and $t = T/Y$, the net profit share is now

$$\pi = 1 - \omega - rd + g - t$$

Introducing a government sector

- A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

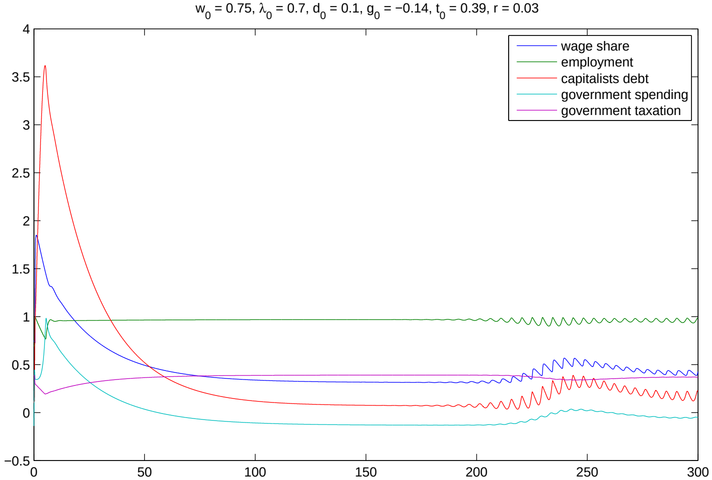
$$\dot{G} = \Gamma(\lambda)Y$$
$$\dot{T} = \Theta(\pi)Y$$

- Defining $g = G/Y$ and $t = T/Y$, the net profit share is now

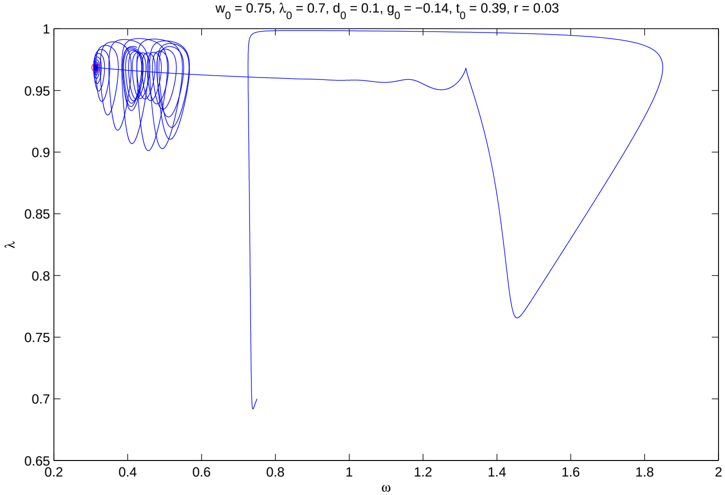
$$\pi = 1 - \omega - rd + g - t$$

- The new 5-dimensional system displays more local fluctuations, but no breakdown for the same initial conditions as before.

Example 5: stabilizing government



Example 5 (continued): stabilizing government



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- Calibrate to macroeconomic time series.

Concluding thoughts

- Solow (1990): The true test of a simple model is whether it helps us to make sense of the world. Marx was, of course, dead wrong about this. We have changed the world in all sorts of ways, with mixed results; the point is to interpret it.

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- Schumpeter (1939): Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them.