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H-theorem for nonlinear Fokker-Planck equations related to generalized thermostatistics. (English summary)
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The authors describe a correspondence principle that associates a nonlinear Fokker-Planck equation (NLFPE) with a generalised entropy functional $\mathcal{S}$, in the sense that stationary solutions for the NLFPE are the distributions for which $\mathcal{S}$ itself is stationary under the constraint of canonical ensembles. In the case where $\mathcal{S}$ is the Shannon entropy, their equation reduces to the linear Fokker-Planck equation, which is used throughout the paper as a reference case.

They then state their main result: If the linear Fokker-Planck equation has a unique stationary solution $\mathcal{W}_{\rm st}(x)$ (a fact which essentially depends on the regularity of the potential that generates the drift term, as is explained in the paper) then the NLFPE associated with $\mathcal{S}$ has a unique stationary solution $\mathcal{P}_{\rm st}(x)$ which can be obtained as a transform of $\mathcal{W}_{\rm st}(x)$ (using the kernel of $\mathcal{S}$). Moreover, the transient solutions for the NLFPE converge to $\mathcal{P}_{\rm st}$ for any arbitrary initial condition $\mathcal{P}_{\rm st}(x,t_0)$.

The proof consists of building up a Lyapunov function out of the entropy functional and then using classical variational calculus arguments (helped by cleverly done integrations by parts) to show convergence and using the uniqueness of $\mathcal{W}_{\rm st}(x)$ together with the fact that the above-mentioned transform is one-to-one to show uniqueness of $\mathcal{P}_{\rm st}(x)$.

The remaining of the paper is dedicated to applying the method to some specific physical models, such as the classical bosons and fermions as proposed by G. Kaniadakis and P. Quarati [Phys. Rev. E **48** (1993), no. 6, 4263–4270].

The conclusion offers interesting remarks about the regimes where the NLFPE reduces to a linear Fokker-Planck equation (when the generalised entropy reduces to the Shannon entropy) as well as the difference between the authors' method of deriving the NLFPE (based on a variational principle) and other approaches (based on a mean field approximation to coupled Langevin equations).

Reviewed by *M. R. Grasselli*