The effect of government in a Keen–Minsky Model

M. R. Grasselli

Sharcnet Chair in Financial Mathematics
Mathematics and Statistics - McMaster University
Joint work with B. Costa Lima, X.-S. Wang, J. Wu

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Goodwin Model (1967) - Assumptions

- Assume that

$$N(t) = N_0 e^{\beta t} \quad \text{(total labour force)}$$
$$a(t) = a_0 e^{\alpha t} \quad \text{(productivity per worker)}$$
$$Y(t) = \nu K(t) = a(t)L(t) \quad \text{(total yearly output)}$$

where K is the total stock of capital and L is the employed population.

- Assume further that

$$\dot{\mathrm{w}} = \Phi(\lambda)\mathrm{w} \quad \text{(Phillips curve)}$$
$$\dot{K} = (Y - \mathrm{w}L) - \delta K \quad \text{(Say's Law)}$$

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}L}{Y} = \frac{\mathrm{w}}{a} \quad \text{(wage share)}$$

$$\lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

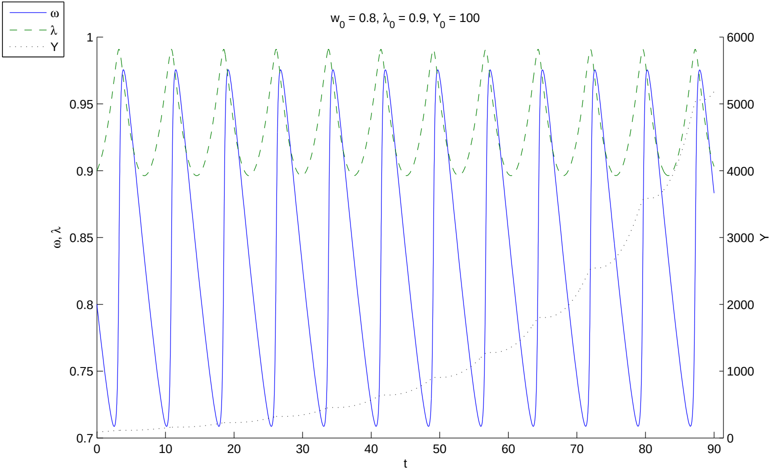
- It then follows that

$$\dot{\omega} = \omega(\Phi(\lambda) - \alpha)$$

$$\dot{\lambda} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \beta - \delta\right)$$

Example 1: Goodwin model

Example 1 (continued): Goodwin model



Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by

$$\dot{K} = \kappa(1-\omega-rd)Y - \delta K$$

where $\kappa(\cdot)$ is $C^1(-\infty,\infty)$ increasing function satisfying certain technical conditions.

- Accordingly, total output evolves as

$$\frac{\dot{Y}}{Y} = \frac{\kappa(1-\omega-rd)}{\nu} - \delta := g(\omega, d)$$

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y$$

Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned} \tag{1}$$

Good equilibrium

- Define

$$\overline{\pi}_1 = \kappa^{-1}(\nu(\alpha+\beta+\delta))$$

- Then the following is an equilibrium for (1):

$$\overline{\omega}_1 = 1 - \overline{\pi}_1 - r\frac{\nu(\alpha+\beta+\delta) - \overline{\pi}_1}{\alpha+\beta}$$

$$\overline{\lambda}_1 = \Phi^{-1}(\alpha)$$

$$\overline{d}_1 = \frac{\nu(\alpha+\beta+\delta) - \overline{\pi}_1}{\alpha+\beta}$$

- Moreover

$$g(\overline{\omega}_1, \overline{d}_1) = \frac{\kappa(1-\overline{\omega}_1 - r\overline{d}_1)}{\nu} - \delta = \alpha+\beta.$$

Bad equilibrium

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

$$
\begin{aligned}
\dot{\omega} &= \omega\,[\Phi(\lambda) - \alpha] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-r/u)}{\nu} - \alpha - \beta - \delta\right] \qquad (2)\\
\dot{u} &= u\left[\frac{\kappa(1-\omega-r/u)}{\nu} - r - \delta\right] - u^2\left[\kappa(1-\omega-r/u) - (1-\omega)\right].
\end{aligned}
$$

- We now see that $(0,0,0)$ is an equilibrium of (2) corresponding to the point

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2) = (0, 0, +\infty)$$

for the original system.

Local stability

- Analyzing the Jacobian of (1) and (2) we obtain the following conclusions.
- The good equilibrium $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1)$ is stable if and only if

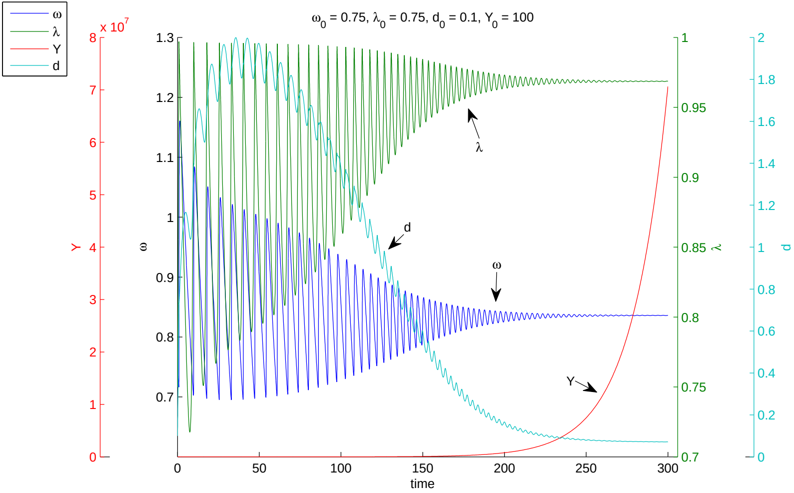
$$r\left[\frac{\kappa'(\overline{\pi}_1)}{\nu}(\overline{\pi}_1 - \kappa(\overline{\pi}_1) + \nu(\alpha+\beta)) - (\alpha+\beta)\right] > 0.$$

- The point $(0,0,0)$ is a stable equilibrium for (2) if and only if

$$\frac{\kappa_0}{\nu} - \delta < r.$$

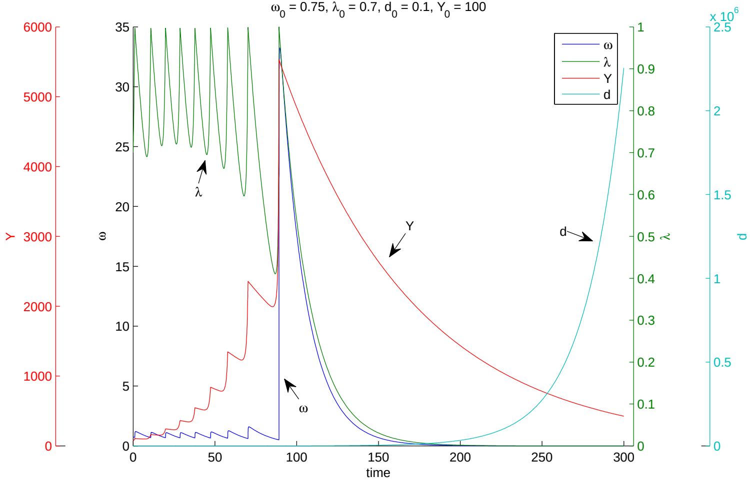
Example 2 : convergence to the good equilibrium in a Keen model

Example 2 (continued): convergence to the good equilibrium in a Keen model

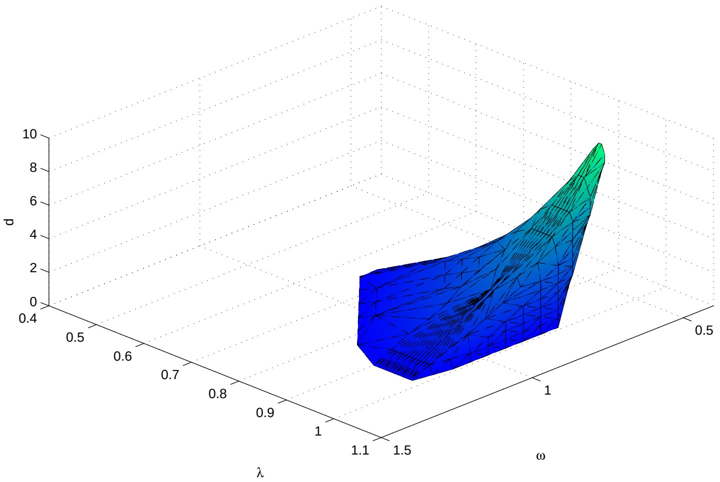


Example 3: explosive debt in a Keen model

Example 3 (continued): explosive debt in a Keen model



Basin of convergence for Keen model



Introducing a government sector

- Following Keen (and echoing Minsky) we add discretionary government spending and taxation into the original system in the form

$$G = G_1 + G_2$$
$$T = T_1 + T_2$$

where

$$\dot{G}_1 = \eta_1(\lambda)Y \qquad \dot{G}_2 = \eta_2(\lambda)G_2$$
$$\dot{T}_1 = \Theta_1(\pi)Y \qquad \dot{T}_2 = \Theta_2(\pi)T_2$$

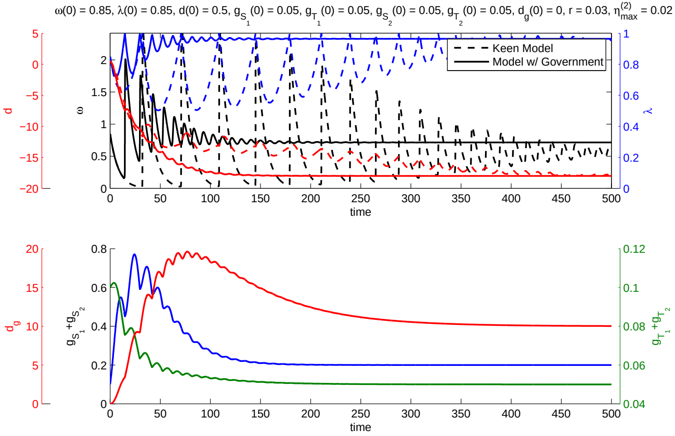
- Defining $g = G/Y$ and $t = T/Y$, the net profit share is now

$$\pi = 1 - \omega - rd + g - t,$$

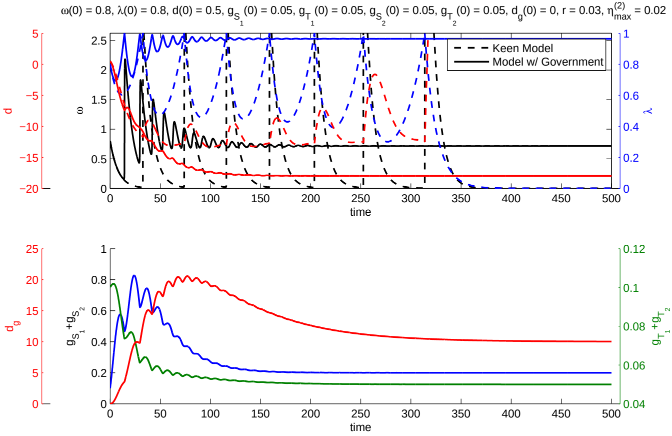
and government debt evolves according to

$$\dot{D}_g = rD_g + G - T.$$

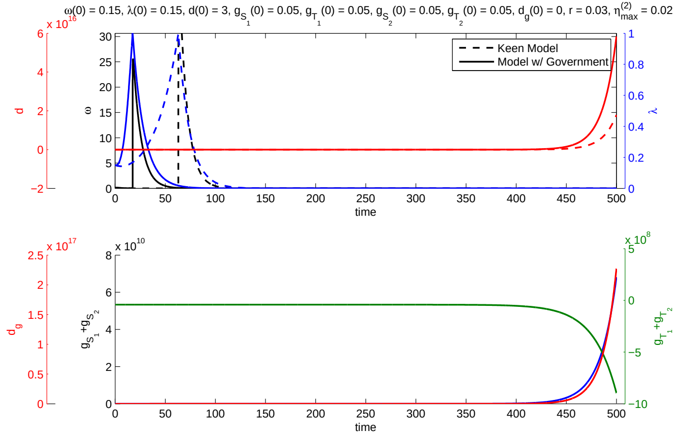
Example 4: Good initial conditions



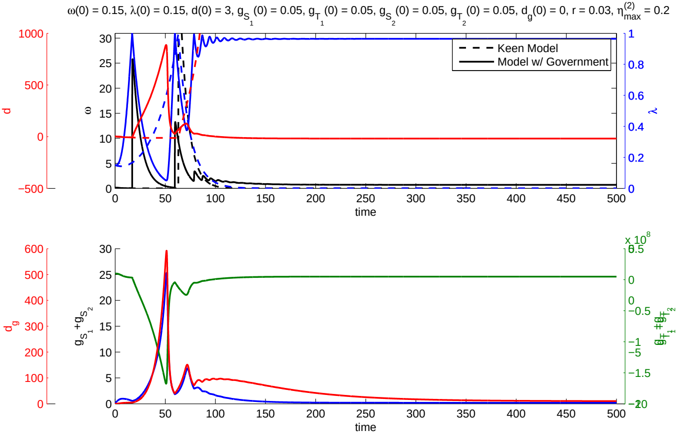
Example 5: Bad initial conditions



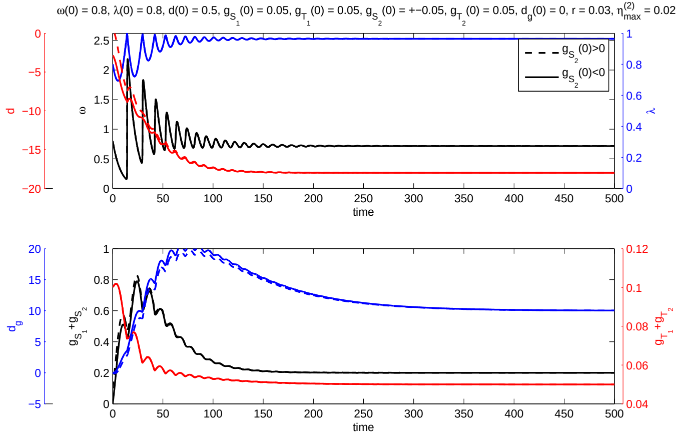
Example 6: Really bad initial conditions with timid government



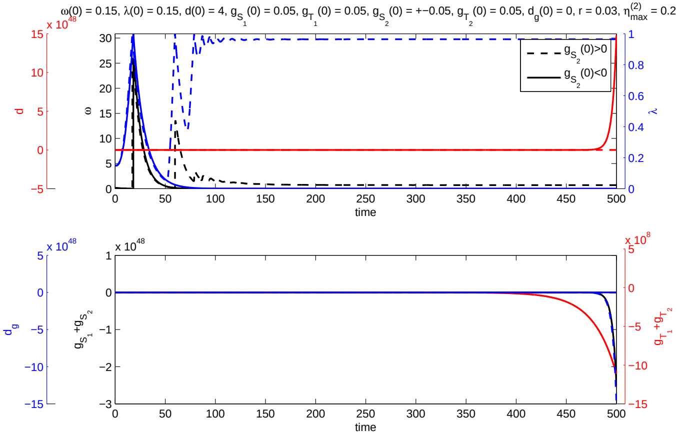
Example 7: Really bad initial conditions with responsive government



Example 8: Austerity in good times: harmless



Example 9: Austerity in bad times: a really bad idea



Next steps

- Model prices for capital goods P_k and commodities P_c explicitly (Kaleckian mark-up theory, inflation, etc)
- Extend the stochastic model (stochastic interest rates, monetary policy, correlated market sectors, etc)
- Extend to an open economy model (exchange rates, capital flows, etc)
- Calibrate to macroeconomic time series