Introduction

Asset Price Bubbles

Modelling Minsky

A dynamical systems model for credit expansion, asset price bubbles and financial fragility

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Dynamic General Equilibrium views

- Seeks to explain the aggregate economy using theories based on strong microeconomic foundations.
- Collective decisions of rational individuals over a range of variables for both present and future.
- All variables are assumed to be simultaneously in equilibrium.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.
- Money is neutral in its effect on real variables.
- Largely ignores uncertainty by simply subtracting risk premia from all risky returns and treat them as risk-free.

Voices of discontent

- M. Morishima (1984): "If economists successfully devise a correct general equilibrium model (...) should it lack the institutional backing to realize an equilibrium solution, then [it] will amount to no more than a utopian state of affairs which bears no relation whatsoever to the real economy."
- A. Kirman (1989): "[DSGE is] empty in the sense that one cannot expect it to house the elements of a scientific theory, one capable of producing empirically falsifiable propositions".
- K. Arrow (1986): "In the aggregate, the hypothesis of rational behavior has in general no implications."
- R. Solow (2006): "Maybe there is in human nature a deep-seated perverse pleasure in adopting and defending a wholly counterintuitive doctrine that leaves the uninitiated peasant wondering what planet he or she is on."

Minsky's alternative interpretation of Keynes

- Neoclassical economics is based on barter paradigm: money is convenient to eliminate the double coincidence of wants.
- In a modern economy, firms make complex portfolios decisions: which assets to hold and how to fund them.
- Financial institutions determine the way funds are available for ownership of capital and production.
- Uncertainty in valuation of cash flows (assets) and credit risk (liabilities) drive fluctuations in real demand and investment.
- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.

Minsky's Financial Instability Hypothesis

- Start when the economy is doing well but firms and banks are conservative.
- Most projects succeed - "Existing debt is easily validated: it pays to lever".
- Revised valuation of cash flows, exponential growth in credit, investment and asset prices.
- Highly liquid, low-yielding financial instruments are devalued, rise in corresponding interest rate.
- Beginning of "euphoric economy": increased debt to equity ratios, development of Ponzi financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
- Euphoria becomes a panic.
- "Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing".

Rational bubbles: definition

- Consider a representative agent solving

$$\sup_c E_t \left[\sum_{j=1}^{\infty} \beta^{j-t} u(c_j) \right]$$

  for exogenously given (e_t, d_t).
- Denoting $q_t = u'(e_t + d_t)p_t$, the FOC for optimality give

$$q_t - \beta E_t[q_{t+1}] = \beta E_t\left[d_{t+1} u'(e_{t+1} + d_{t+1})\right]$$

- The general solution is of the form $q_t = F_t + B_t$ where

$$F_t = \sum_{j=1}^{\infty} \beta^j E_t\left[d_{t+j} u'(e_{t+j} + d_{t+j})\right]$$

  is the fundamental price and B_t is a bubble term satisfying

$$E_t[B_{t+1}] = \beta^{-1} B_t \tag{1}$$

Consequences

- $B_t \geq 0$ for all t.
- Any nonzero rational bubble must start with $B_0 > 0$.
- If $T < \infty$, $B_t = 0$ for all $0 \leq t \leq T$, and this result is robust with respect to diverse information (Tirole 1982).
- If $T = \infty$, bubbles can exit in a myopic rational expectations equilibrium.
- Rational bubbles cannot exist in a fully dynamic REE with finitely many infinitely lived agents.
- They can exit in an overlapping generations models provided $0 < \bar{r} < g$, where $\bar{r}$ is the asymptotic real interest rate and g is the rate of growth of the economy (Tirole 1985).

The Efficient Markets Hypothesis

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- Denote $R_{t+1} = \frac{p_{t+1} - p_t + d_{t+1}}{p_{t+1}}$.
- As we have seen, a first-order rational expectations condition for risk-neutral agents leads to

$$E_t[R_{t+1}] = 1 + r. \tag{2}$$

- Solving this recursively leads to

$$p_t = \sum_{j=1}^{\infty} \frac{1}{(1+r)^j} E_t[d_{t+j}], \tag{3}$$

plus a possible rational bubble term.
- Either (2) or (3) can be taken as an EMH.
- Statistical tests on actual returns indicate that they are not *very* forecastable, leading to the conclusion that the EMH cannot be rejected.
- Nevertheless, something seems very wrong...

Excess volatility of stock prices (Shiller, 1981)

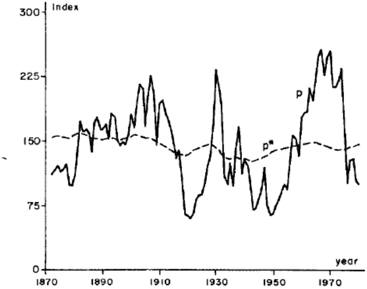

FIGURE 1

Note: Real Standard and Poor's Composite Stock Price Index (solid line p) and *ex post* rational price (dotted line p^*), 1871–1979, both detrended by dividing a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter. Data are from Data Set 1, Appendix.

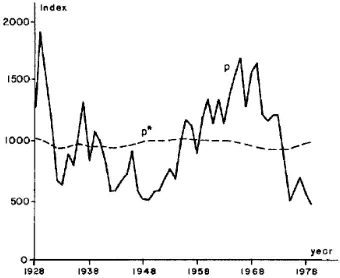

FIGURE 2

Note: Real modified Dow Jones Industrial Average (solid line p) and *ex post* rational price (dotted line p^*), 1928-1979, both detrended by dividing by a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter Data are from Data Set 2, Appendix.

Alternative models (Shiller, 1984)

- Consider a model where sophisticated investors have a demand function (portion of shares) of the form

$$Q_t^i = \frac{E_t[R_{t+1}] - \alpha}{\phi}. \tag{4}$$

- In addition, suppose there are noise traders who react to fads Y_t through a demand function $Q_t^n = Y_t/p_t$.
- In equilibrium we have $Q_t + \frac{Y_t}{p_t} = 1$.
- Inserting this into (4) and solving recursively leads to

$$p_t = \sum_{j=1}^{\infty} \frac{E_t[d_{t+j}] + \phi E_t[Y_{t-1+j}]}{(1+\alpha+\phi)^j}. \tag{5}$$

- This is also consistent with prices being not very forecastable.

Other sources of inefficiencies

- Noise trader risk (DeLong, Shleifer, Summers and Waldmann 1990): prices deviate from fundamental value because of uncertainty created by noise traders, who can in some cases earn higher expected returns than sophisticated investors.
- Limits of arbitrage (Shleifer and Vishny 1997): performance based arbitrage lead to fund managers leaving the market exactly when they are needed to restore fundamental value.
- No short-sales and diverse beliefs (Miller 1977, Harrison and Kreps 1978): pessimists stay on sidelines and optimists overbid leading to prices higher than fundamentals.
- Overconfidence (Scheinkman and Xiong 2003): mean reverting confidence levels lead to prices that contain an option to re-sell the asset at a later time.

Financial Intermediation (Allen and Gale, 2000)

- Risk-neutral investors with no wealth and banks with $B > 0$ funds to lend at rate r trading at $t = 1, 2$.
- Safe asset (s) with return $(1 + r)$ and a risky asset (R) with price at $t = 2$ given by a random variable p_2 with density $h(p_2)$ on $[0, p_2^{\max}]$ and mean $\overline{p_2}$.
- The equilibrium price in the presence of banks is then

$$p_1 = \frac{1}{1+r}\left[\frac{\int_{(1+r)p_1}^{p_2^{\max}} p_2 h(p_2)dp_2 - c'(1)}{\text{Prob}[p_2 \geq (1+r)p_1]}\right]. \qquad (6)$$

- Define the fundamental value as the price that an investor would pay if he had to use his own money $B > 0$.
- This leads to

$$p_1^F = \frac{\overline{p_2} - c'(1)}{1+r}. \qquad (7)$$

- It can shown that $p_1 \geq p_1^F$.

Basic Goodwin Model (1982)

- Let

$$N(t) = N_0 e^{\beta t} \qquad \text{(labour force)}$$
$$a(t) = a_0 e^{\alpha t} \qquad \text{(productivity per worker)}$$
$$\lambda = L/N \qquad \text{(employment rate)}$$

- Assume that wages satisfy

$$\mathrm{w}' = F_w(\lambda)\mathrm{w},$$

where $F_w(\lambda)$ is a Phillips curve.

- Denoting the wage share of the total economic output by

$$\omega(t) := \frac{\mathrm{w}(t)L(t)}{a(t)L(t)} = \frac{\mathrm{w}(t)}{a(t)},$$

it then follows that

$$\frac{\omega'}{\omega} = \frac{a}{\mathrm{w}}\left(\frac{\mathrm{w}}{a}\right)' = F_{\mathrm{w}}(\lambda) - \alpha.$$

Basic Goodwin Model (continued)

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- Define

$$Y(t) = a(t)L(t) \qquad \text{(total yearly output)}$$
$$K(t) = \nu Y(t) \qquad \text{(total capital)}$$

- Suppose further that the rate of change in capital is

$$K' = I_g(t) - \gamma K = (1-\omega)Y - \gamma K$$

- It then follows that

$$\frac{\lambda'}{\lambda} = \frac{N}{L}\left(\frac{L}{N}\right)' = \frac{aN}{Y}\left(\frac{Y}{aN}\right)' = \frac{1-\omega}{\nu} - \gamma - \alpha - \beta$$

Basic Goodwin Model (summary)

- We see that the basic Goodwin model reduces to the following version of a predator-prey dynamical system:

$$\omega' = \omega(F_w(\lambda) - \alpha) \tag{8}$$

$$\lambda' = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \gamma - \beta\right) \tag{9}$$

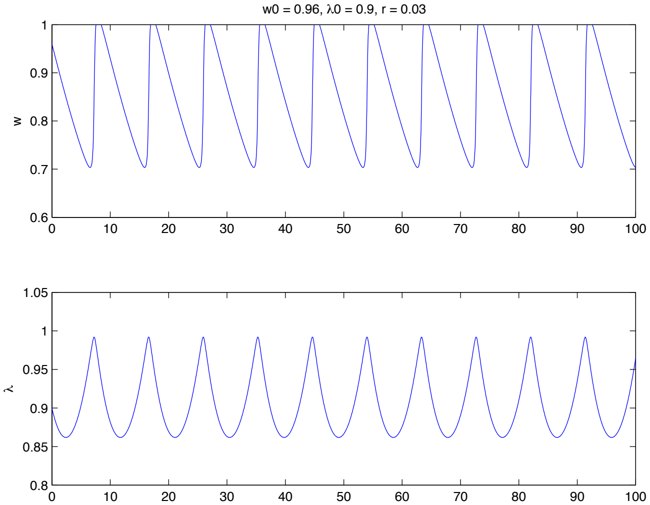
- The unique (non-zero) equilibrium point is

$$\lambda_e = F_w^{-1}(\alpha)$$
$$\omega_e = 1 - \nu(\gamma + \alpha + \beta)$$

- This system is globally stable and leads to endogenous business cycles.

Example 1: basic Goodwin model

Example 1 (continued): basic Goodwin model



Financial sector (Keen, 1995)

- Introduce now a banking sector to finance new investment through an amount of debt evolving according to

$$D' = rD + I_g - (1-\omega)Y.$$

- Denote the banking share of the economy by $d = D/Y$ and define the net profit share as

$$\pi_n(t) = 1 - \omega(t) - rd(t).$$

- The next key assumption is that gross new investment is given by

$$I_g := \kappa\left(\frac{\pi_n}{\nu}\right)Y,$$

where $\kappa(\cdot)$ is an increasing nonlinear function.
- That is, the rate of change in capital is now

$$K' = I_g(t) - \gamma K = \kappa\left(\frac{\pi_n}{\nu}\right)Y - \gamma K$$

Differential Equations

- We then have that

$$\frac{Y'}{Y} = \frac{1}{Y}\left(\frac{K}{\nu}\right)' = \frac{\kappa\left(\frac{1-\omega-rd}{\nu}\right)}{\nu} - \gamma =: F_Y(\omega, d).$$

- The exact same calculation as before gives that

$$\frac{\lambda'}{\lambda} = F_Y(\omega, d) - \alpha - \beta$$

- On the other hand, it follows that the evolution of debt is given by

$$\begin{aligned} d' = \left(\frac{D}{Y}\right)' &= \frac{D'}{Y} - \left(\frac{D}{Y}\right)\frac{Y'}{Y} \\ &= rd + \kappa(\pi_n/\nu) - (1-\omega) - dF_Y(\omega, d) \end{aligned} \tag{10}$$

Keen model of private debt (summary)

- The corresponding dynamical systems now reads

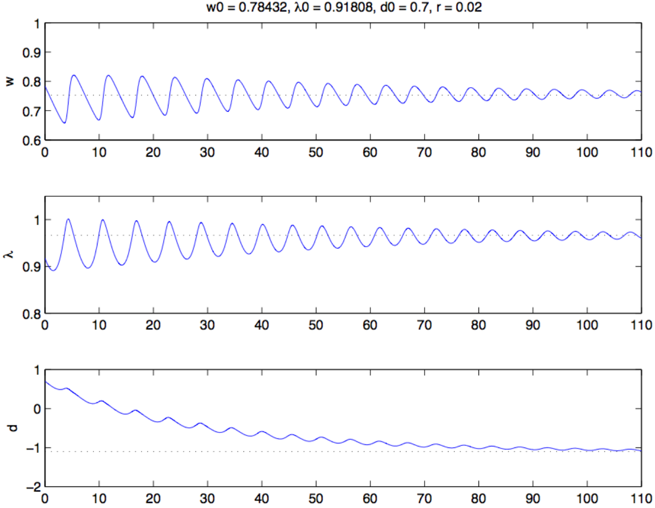
$$\begin{aligned} \omega' &= \omega(F_w(\lambda) - \alpha) \\ \lambda' &= \lambda\left(F_Y(\omega, d) - \alpha - \beta\right) \\ d' &= d[r - F_Y(\omega, d)] + \nu[F_Y(\omega, d) + \gamma] - (1 - \omega) \end{aligned}$$

- This system is locally stable but globally unstable.
- One possible equilibrium point is given by

$$\begin{aligned} \lambda_e &= F_w^{-1}(\alpha) \\ \omega_e &= 1 - rd_e - \pi_e \\ d_e &= \frac{\kappa(\pi_e/\nu) - \pi_e}{\frac{\kappa(\pi_e/\nu)}{\nu} - \gamma} \end{aligned}$$

where $\pi_e = \nu\kappa^{-1}(\nu(\gamma + \alpha + \beta))$.

Example 2: convergent Goodwin model with banks



Example 2 (continued): convergent Goodwin model with banks

Other equilibria

- A second equilibrium occurs when $\omega = \lambda = 0$ and d solves the equation

$$d_2 = \frac{\kappa((1 - rd)/\nu) - 1 + rd}{\kappa((1 - rd)/\nu)/\nu - \gamma}.$$

- A third equilibrium consists of $\omega = 0$, $\lambda \in \mathbb{R}^+$ and d satisfying

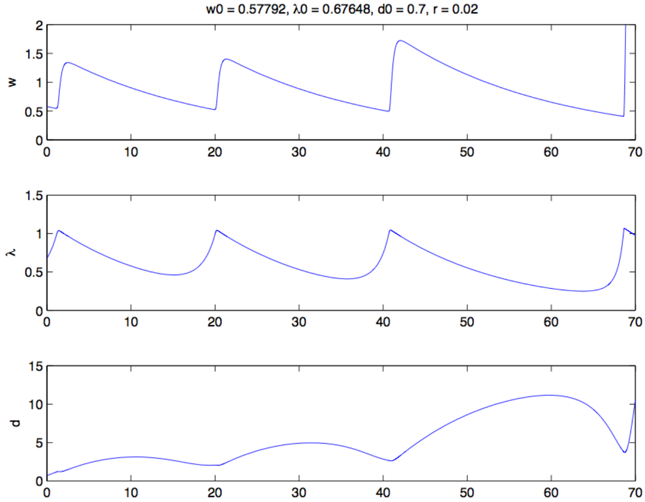
$$d_3 = \frac{\nu((\alpha + \beta + \gamma) - \kappa^{-1}(\nu(\alpha + \beta + \gamma)))}{\alpha + \beta}$$

- Finally, rewriting the system in terms of $(\omega, \lambda, 1/d)$ we see that a fourth equilibrium of the original system consists of

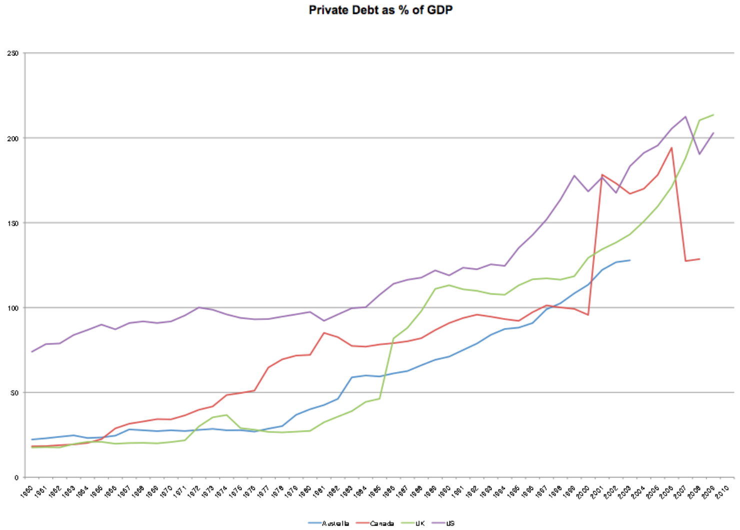
$$(\omega = 0, \lambda = 0, d = \infty)$$

- Both the first and last equilibria are stable for a wide range of parameters.

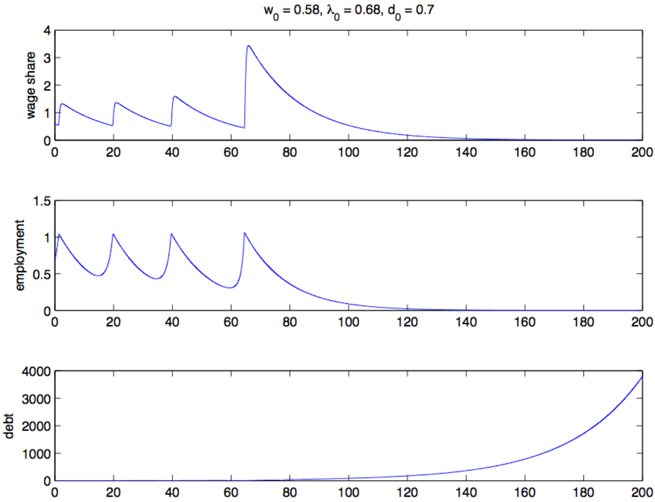
Example 3: divergent Goodwin model with banks



Detour: a bit of data



Example 3 (continued): divergent Goodwin model with banks



Example 3 (continued): divergent Goodwin model with banks

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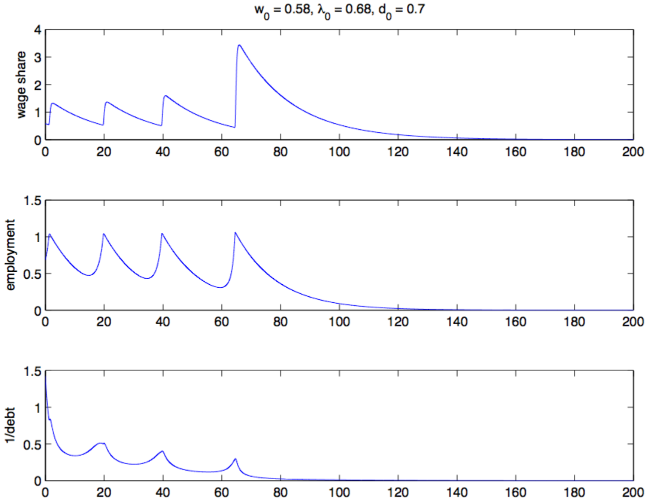
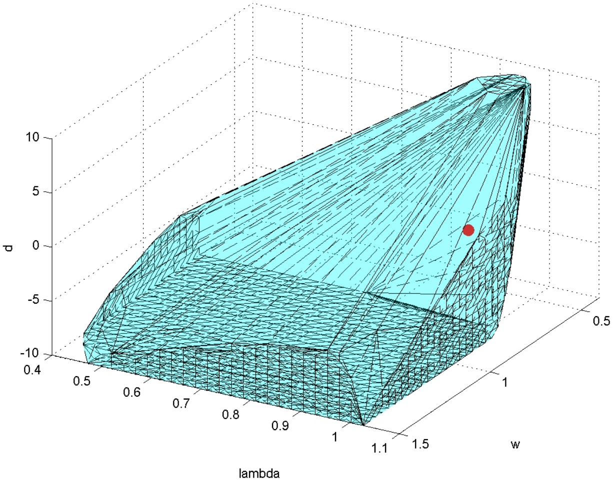


Example 3 (continued): divergent Goodwin model with banks

Basin of convergence for Goodwin model with banks



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- To introduce the destabilizing effect of purely speculative investment consider a modified version of the previous model with

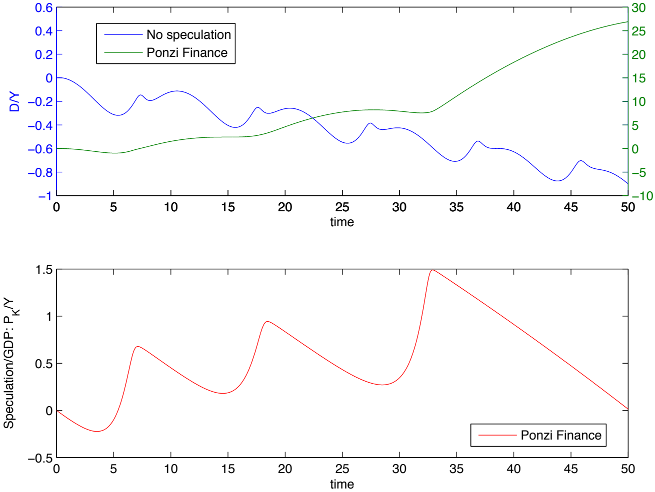
$$D' = I_g - \pi_n + P_k,$$

where

$$P_k' = F_p(F_Y)Y$$

- Here $F_p(\cdot)$ is a increasing nonlinear function of the growth rate of economic output F_Y.

Effect of Ponzi financing



Introducing a government sector

- A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

$$G' = F_G(\gamma)Y$$
$$T' = F_T(\pi)Y$$

- Defining $g = G/Y$ and $t = T/Y$, the net profit share is now

$$\pi_n = 1 - \omega - t + g - rd$$

- The new 5-dimensional system displays more local fluctuations, but no breakdown for all tested cases.
- Interestingly, the equilibrium value for government debt exhibits an explicit bifurcation r.

Next steps

- Model prices for capital goods P_k and commodities P_c explicitly (Kaleckian mark-up theory, inflation, etc)
- Introduce noise (stochastic interest rates, risk premium, etc)
- Calibrate to macroeconomic time series.
- Thanks !