On the optimal exercise policy for executive stock options

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1. A binomial model for real options

Consider a one-period model with \( \Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\} \), and historical probabilities \( P\{\omega_i\} = p_i > 0 \) such that

\[
\begin{align*}
S_T(\omega_1) &= uS_0, & Y_T(\omega_1) &= hY_0, \\
S_T(\omega_2) &= uS_0, & Y_T(\omega_2) &= \ell Y_0, \\
S_T(\omega_3) &= dS_0, & Y_T(\omega_3) &= hY_0, \\
S_T(\omega_4) &= dS_0, & Y_T(\omega_4) &= \ell Y_0,
\end{align*}
\]

where \( 0 < d < 1 < u \) and \( 0 < \ell < 1 < h \), for positive initial values \( S_0, Y_0 \).
Let $B$ be a contingent claim on $Y$. If we denote

$$B_h = B_T(\omega_1) = B_T(\omega_3) = B(hY_0)$$
$$B_\ell = B_T(\omega_2) = B_T(\omega_4) = B(\ell Y_0),$$

then its (exponential utility) indifference price is

$$\pi^B = -\frac{1}{\gamma} \left( q \log \left[ \frac{e^{-\gamma B_h p_1} + e^{-\gamma B_\ell p_2}}{p_1 + p_2} \right] + (1 - q) \log \left[ \frac{e^{-\gamma B_h p_3} + e^{-\gamma B_\ell p_4}}{p_3 + p_4} \right] \right), \quad (1)$$

where

$$q = \frac{1 - d}{u - d}.$$
Now suppose $B$ is an American claim. It is clear that early exercise will occur whenever

$$B(Y_0) \geq \pi^B,$$

where $\pi^B$ is the (European) indifference price. For example, an American call option with strike price $K$ will be exercised if $Y_0$ exceeds the solution to

$$Y^* - K = \log \left[ \left( \frac{p_1 + p_2}{e^{-\gamma B_h p_1} + e^{-\gamma B_\ell p_2}} \right)^{\frac{q}{\gamma}} \left( \frac{p_3 + p_4}{e^{-\gamma B_h p_3} + e^{-\gamma B_\ell p_4}} \right)^{\frac{1-q}{\gamma}} \right].$$
2. **Multiple claims**

As a result of risk aversion, the early exercise threshold for an American call option obtained above is different (and higher) than the exercise threshold for a contract consisting of $A$ units of identical American calls.
If partial exercise is allowed, then the optimal number of options to be exercised is the solution $a^*$ to

$$\max_a \left[ a(Y_0 - K)^+ + \pi^{(A-a)B} \right].$$

(2)
3. **Multiperiod: inter-temporal exercise**

- Begin at the final period.

- At each node of the tree, compute the (European) indifference prices for different values of \((A - a)\).

- Determining the maximum of \((2)\).

- Use this as the value for the entire position at that node.

- Iterate backwards.