On the optimal exercise policy for executive stock options

M. R. Grasselli V. Henderson

DQF, Cambridge
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1. A binomial model for real options

Consider a one-period model with $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, and historical probabilities $P\{\omega_i\} = p_i > 0$ such that

$$\begin{aligned} S_T(\omega_1) &= uS_0, \qquad & Y_T(\omega_1) = hY_0, \\ S_T(\omega_2) &= uS_0, \qquad & Y_T(\omega_2) = \ell Y_0, \\ S_T(\omega_3) &= dS_0, \qquad & Y_T(\omega_3) = hY_0, \\ S_T(\omega_4) &= dS_0, \qquad & Y_T(\omega_4) = \ell Y_0, \end{aligned}$$

where $0 < d < 1 < u$ and $0 < \ell < 1 < h$, for positive initial values S_0, Y_0.

Let B be a contingent claim on Y. If we denote

$$\begin{aligned} B_h &= B_T(\omega_1) = B_T(\omega_3) = B(hY_0) \\ B_\ell &= B_T(\omega_2) = B_T(\omega_4) = B(\ell Y_0), \end{aligned}$$

then its (exponential utility) indifference price is

$$\begin{aligned} \pi^B &= -\frac{1}{\gamma}\left(q \log\left[\frac{e^{-\gamma B_h}p_1 + e^{-\gamma B_\ell}p_2}{p_1+p_2}\right] + \right. \\ & \left. (1-q)\log\left[\frac{e^{-\gamma B_h}p_3 + e^{-\gamma B_\ell}p_4}{p_3+p_4}\right]\right), \end{aligned} \tag{1}$$

where

$$q = \frac{1-d}{u-d}.$$

Now suppose B is an American claim. It is clear that early exercise will occur whenever

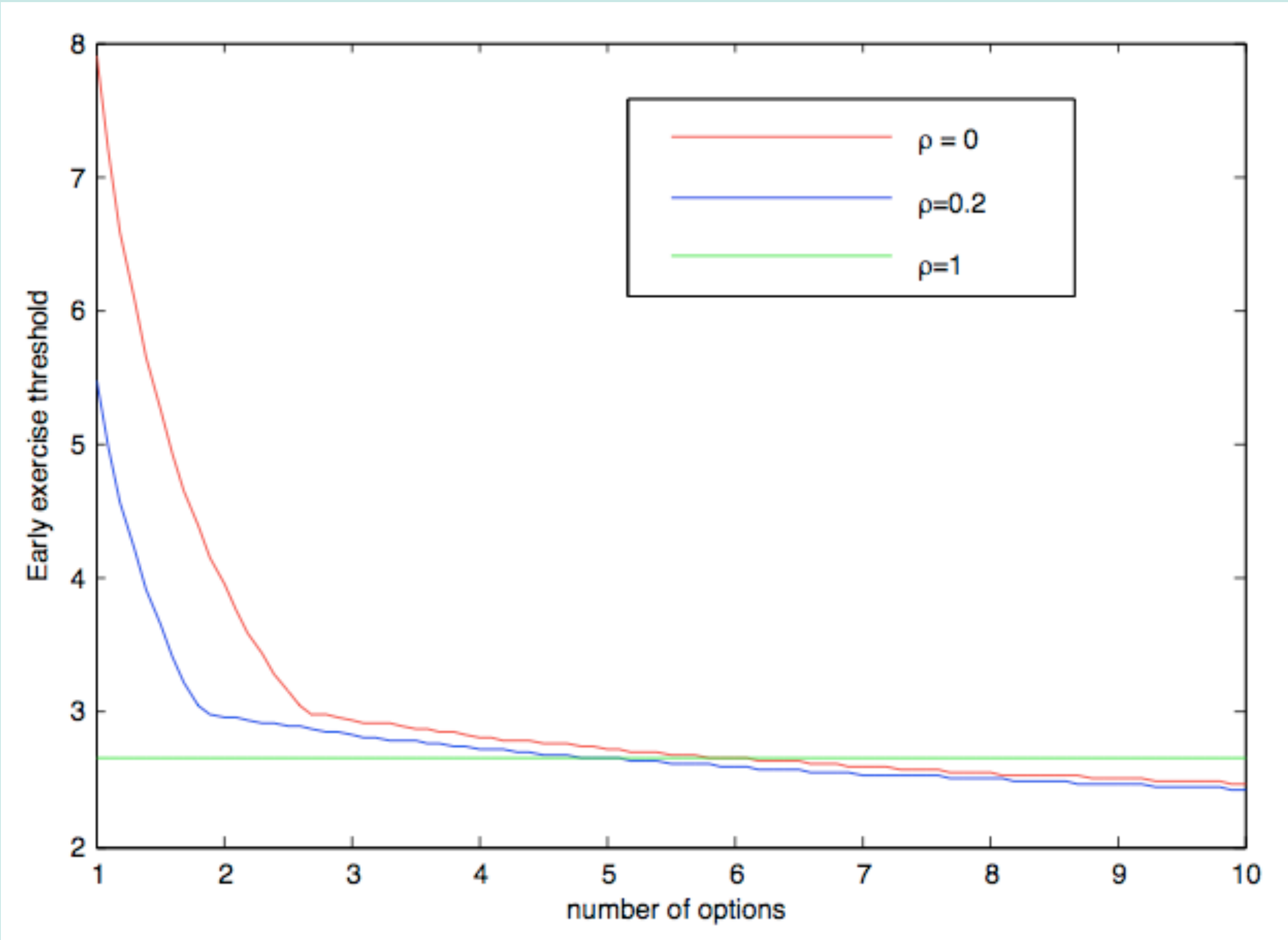
$$B(Y_0) \geq \pi^B,$$

where π^B is the (European) indifference price. For example, an American call option with strike price K will be exercised if Y_0 exceeds the solution to

$$Y^* - K = \log\left[\left(\frac{p_1 + p_2}{e^{-\gamma B_h} p_1 + e^{-\gamma B_\ell} p_2}\right)^{\frac{q}{\gamma}} \left(\frac{p_3 + p_4}{e^{-\gamma B_h} p_3 + e^{-\gamma B_\ell} p_4}\right)^{\frac{1-q}{\gamma}}\right].$$

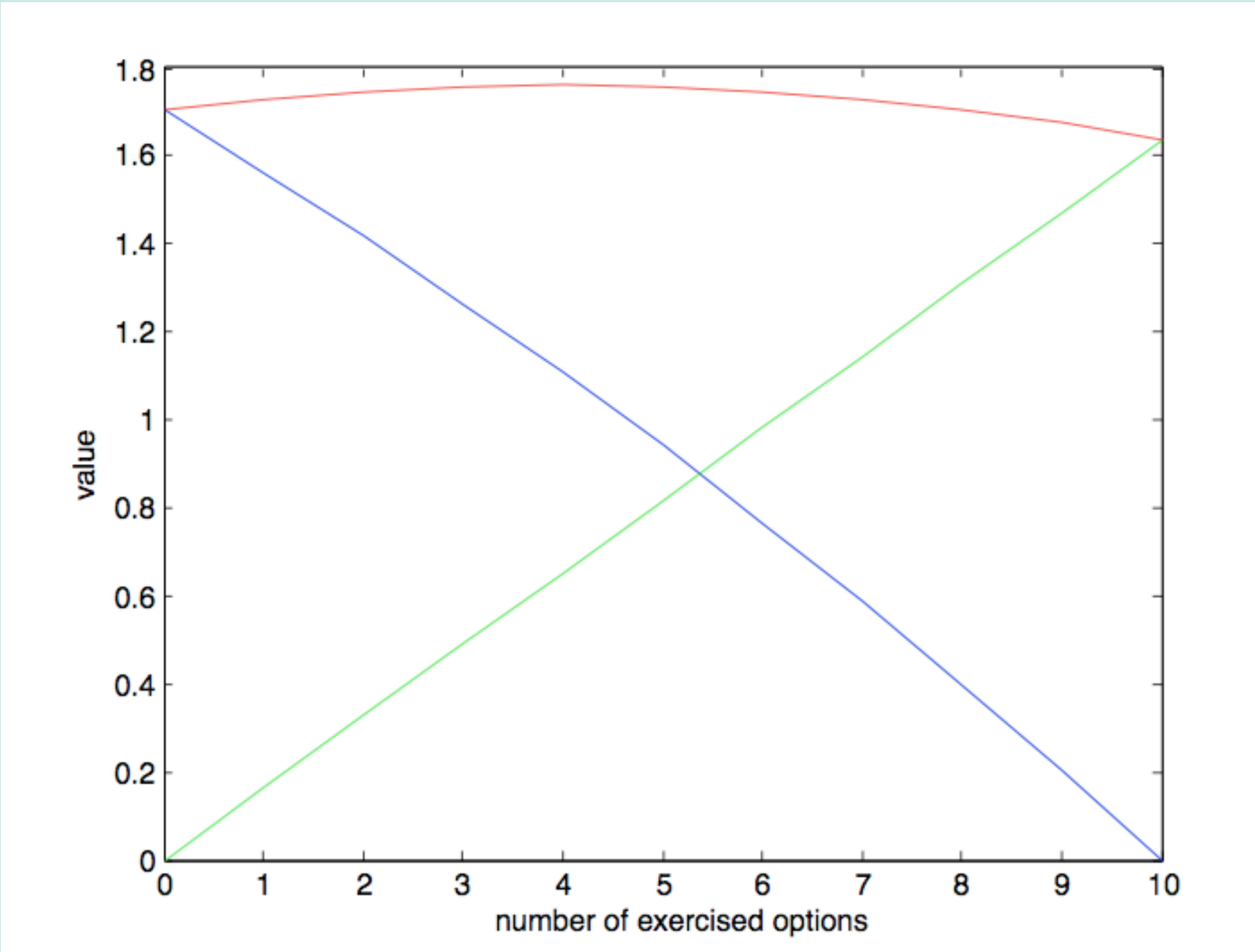
2. Multiple claims

As a result of risk aversion, the early exercise threshold for an American call option obtained above is different (and higher) than the exercise threshold for a contract consisting of A units of identical Americal calls.


Early exercise threshold
number of options
ρ = 0
ρ=0.2
ρ=1
8
7
6
5
4
3
2
1
2
3
4
5
6
7
8
9
10

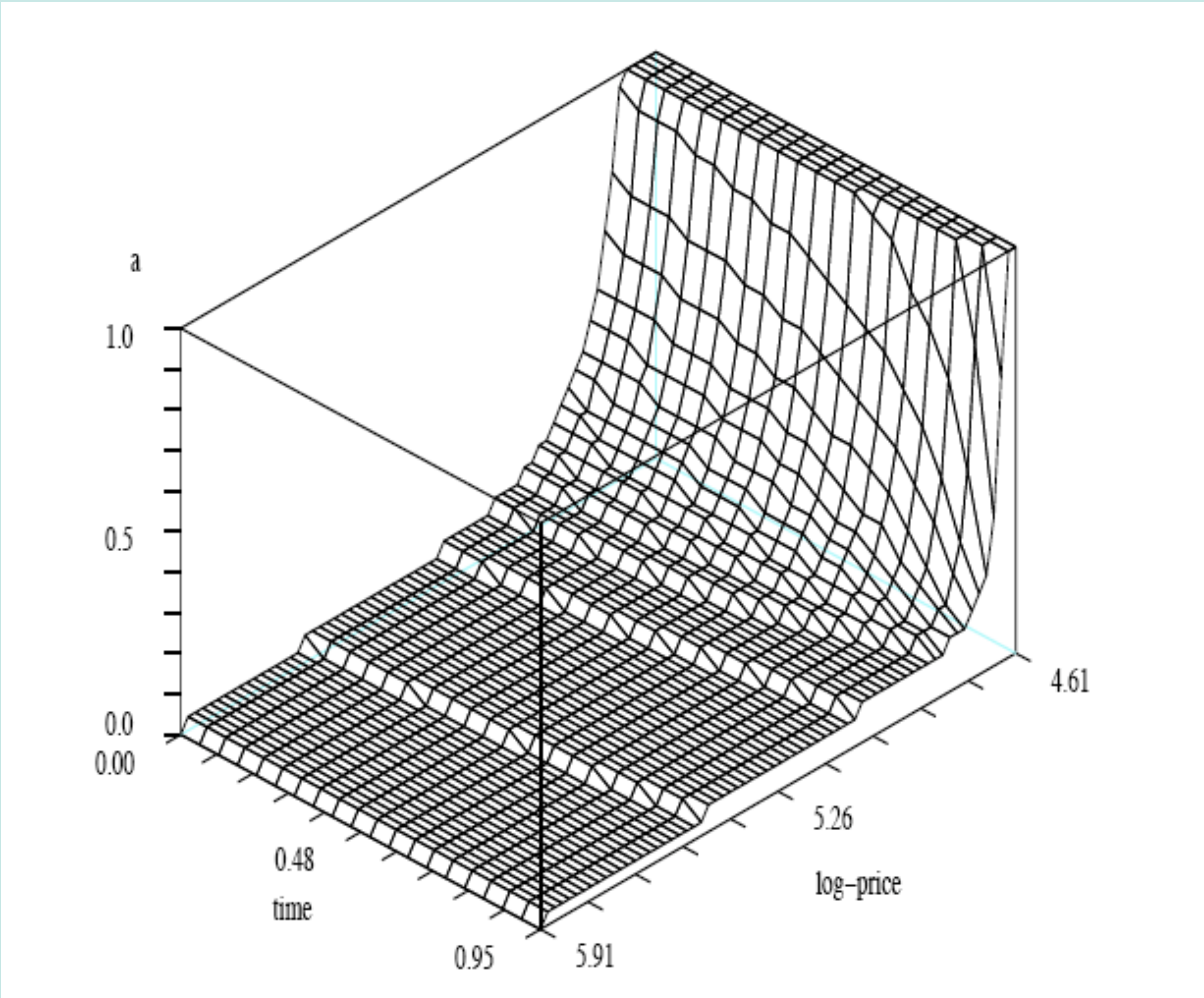
If partial exercise is allowed, then the optimal number of options to be exercised is the solution a^* to

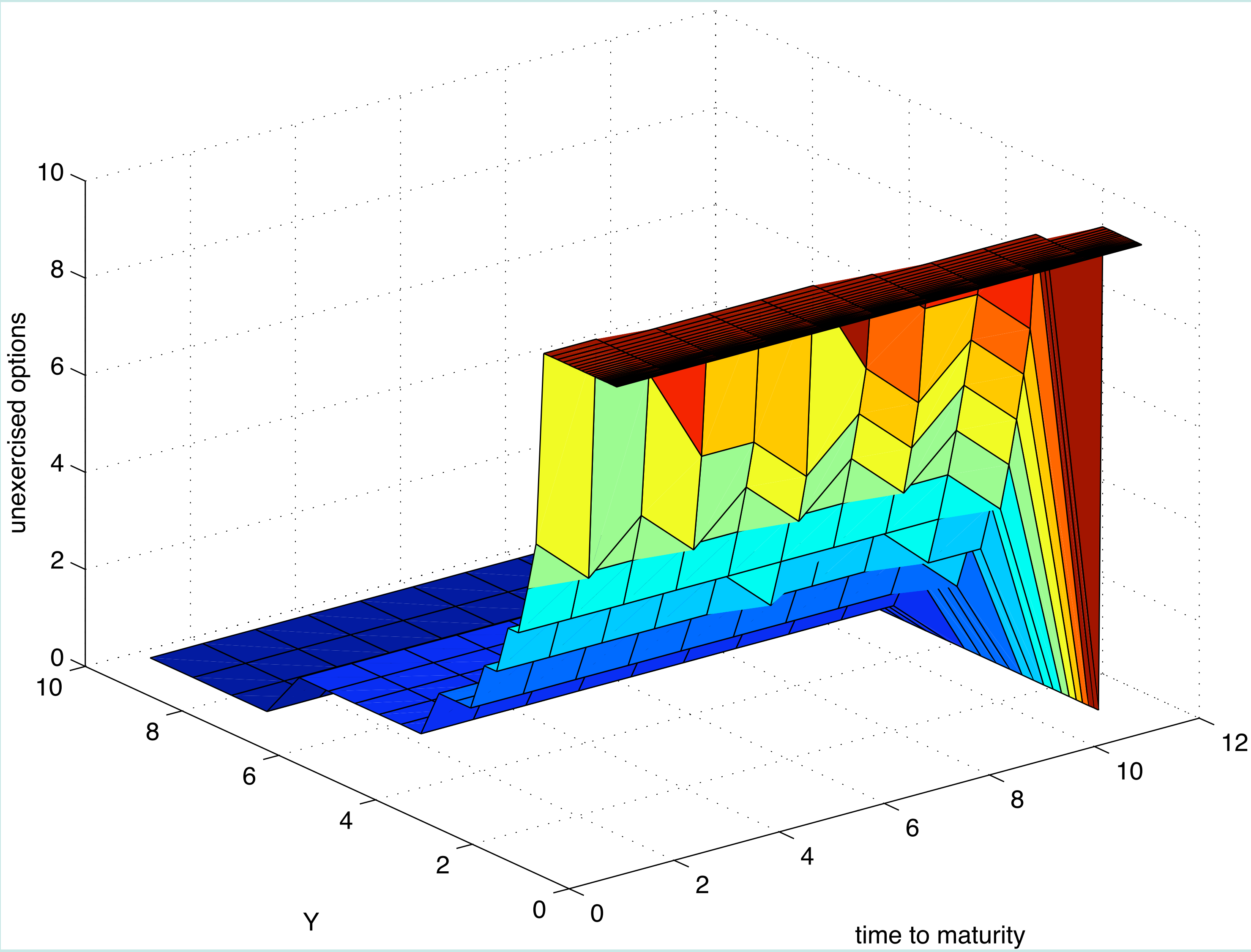
$$\max_a \left[a(Y_0 - K)^+ + \pi^{(A-a)B} \right]. \tag{2}$$

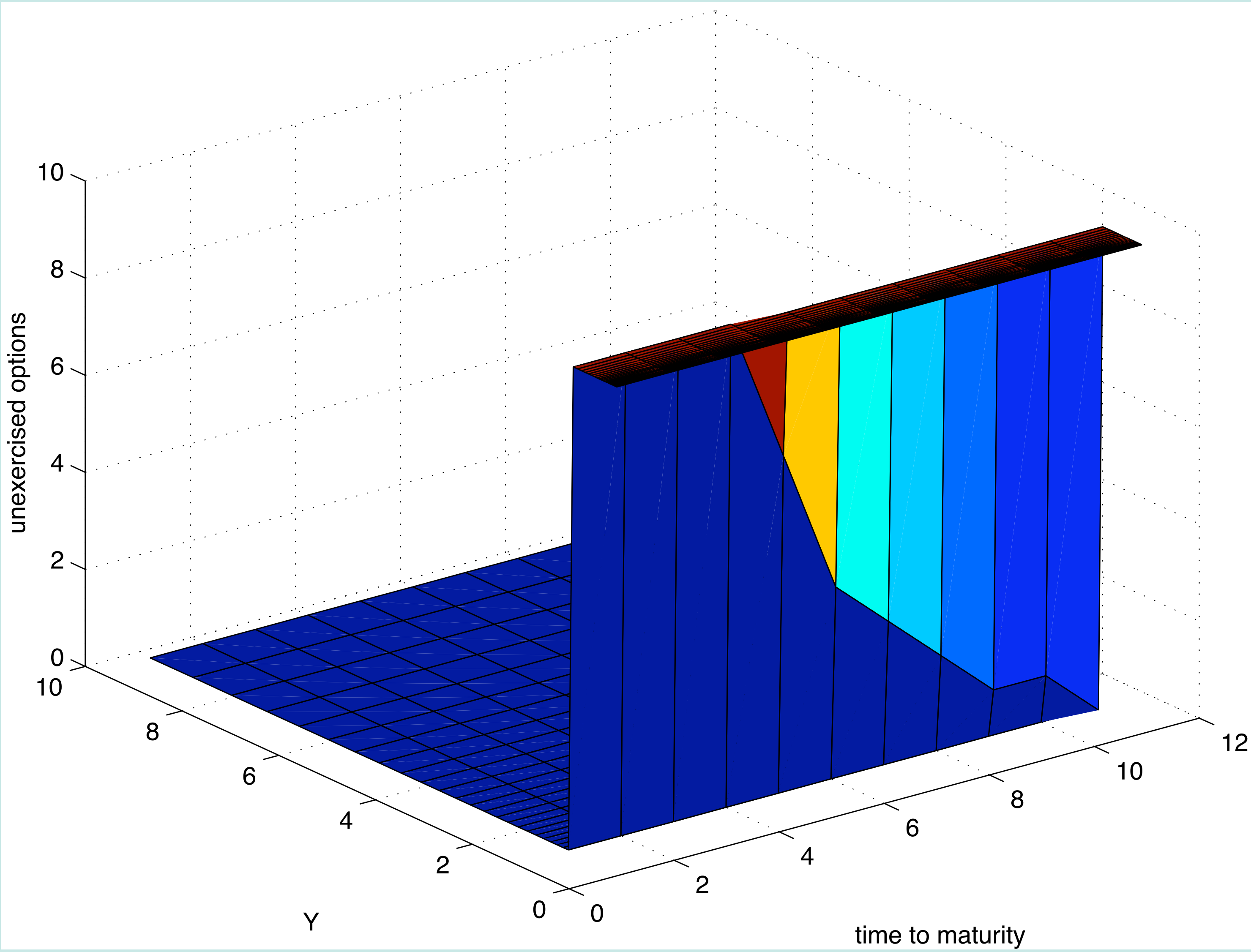

1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
value
0
1
2
3
4
5
6
7
8
9
10
number of exercised options

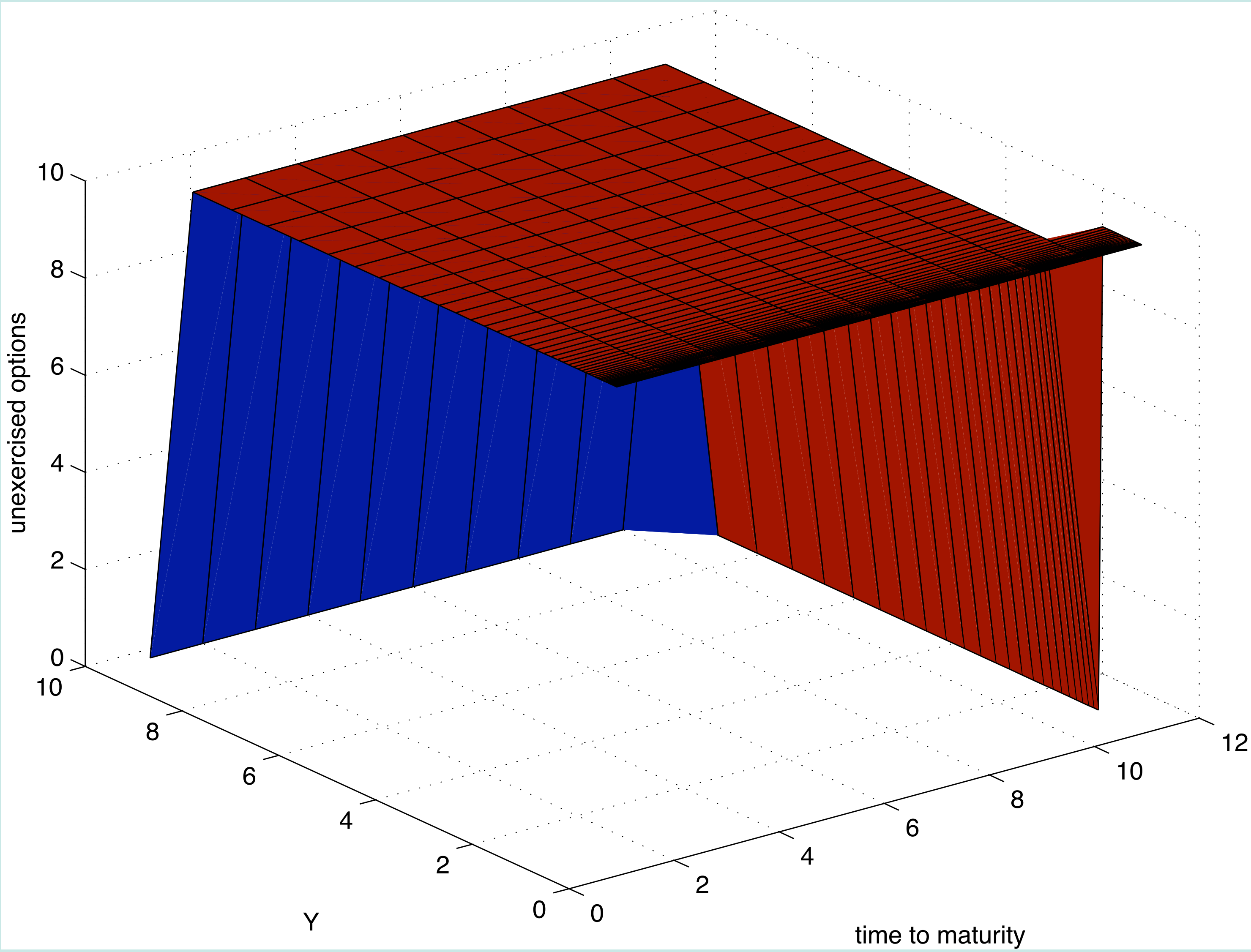
3. Multiperiod: inter-temporal exercise

- Begin at the final period.
- At each node of the tree, compute the (European) indifference prices for different values of $(A - a)$.
- Determining the maximum of (2).
- Use this as the value for the entire position at that node.
- Iterate backwards.


a
1.0
0.5
0.0
0.00
0.48
time
0.95
5.91
5.26
log-price
4.61


unexercised options
0
2
4
6
8
10
Y
0
2
4
6
8
10
time to maturity
0
2
4
6
8
10
12


unexercised options
10
8
6
4
2
0
10
8
6
4
2
0
Y
0
2
4
6
8
10
12
time to maturity



unexercised options
0
2
4
6
8
10
Y
time to maturity
12