A Stochastic Extension of the Keen-Minsky Model for Financial Fragility

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Outline

Minsky's Financial Instability Hypothesis

- Start when the economy is doing well but firms and banks are conservative.
- Most projects succeed - "Existing debt is easily validated: it pays to lever".
- Revised valuation of cash flows, exponential growth in credit, investment and asset prices.
- Highly liquid, low-yielding financial instruments are devalued, rise in corresponding interest rate.
- Beginning of "euphoric economy": increased debt to equity ratios, development of Ponzi financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
- Euphoria becomes a panic.
- "Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing".

Goodwin Model (1967) - Assumptions

- Assume that

$$\begin{aligned} N(t) &= N_0 e^{\beta t} && \text{(total labour force)} \\ a(t) &= a_0 e^{\alpha t} && \text{(productivity per worker)} \\ Y(t) &= \nu K(t) = a(t)L(t) && \text{(total yearly output)} \end{aligned}$$

  where K is the total stock of capital and L is the employed population.

- Assume further that

$$\begin{aligned} \dot{\mathrm{w}} &= \Phi(\lambda)\mathrm{w} && \text{(Phillips curve)} \\ \dot{K} &= (Y - \mathrm{w}L) - \delta K && \text{(Say's Law)} \end{aligned}$$

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}L}{Y} = \frac{\mathrm{w}}{a} \quad \text{(wage share)}$$

$$\lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

- It then follows that

$$\dot{\omega} = \omega(\Phi(\lambda) - \alpha)$$

$$\dot{\lambda} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \beta - \delta\right)$$

Goodwin Model - Properties

- If we take Φ to be linear, this is a predator-prey model.
- To ensure $\lambda \in (0,1)$ we assume instead that Φ is $C^1(0,1)$ and satisfies

$$
\begin{aligned}
&\Phi'(\lambda) > 0 \text{ on } (0,1)\\
&\Phi(0) < \alpha\\
&\lim_{\lambda \to 1^-} \Phi(\lambda) = \infty.
\end{aligned}
$$

- Then $(\overline{\omega}_0, \overline{\lambda}_0) = (0,0)$ is a saddle point and the only other equilibrium

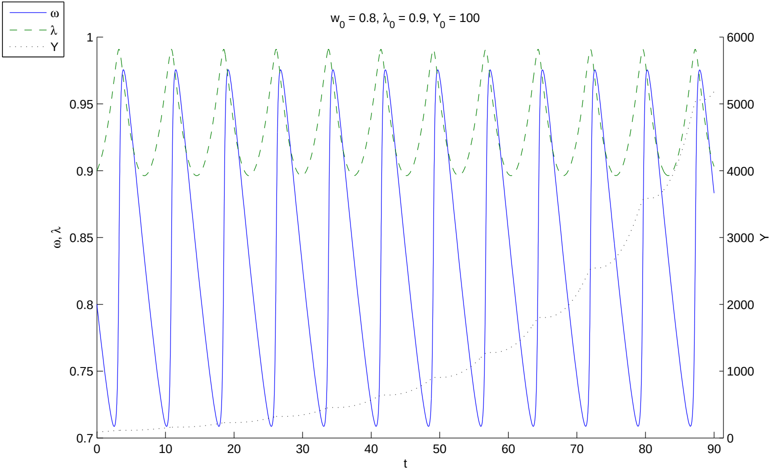
$$(\overline{\omega}_1, \overline{\lambda}_1) = \left(1 - \nu(\alpha+\beta+\delta), \Phi^{-1}(\alpha)\right)$$

is non-hyperbolic.
- Moreover

$$g(\overline{\omega}_1) := \frac{\dot{Y}}{Y}(\overline{\omega}_1) = \frac{1-\overline{\omega}_1}{\nu} - \delta = \alpha + \beta,$$

Example 1: Goodwin model

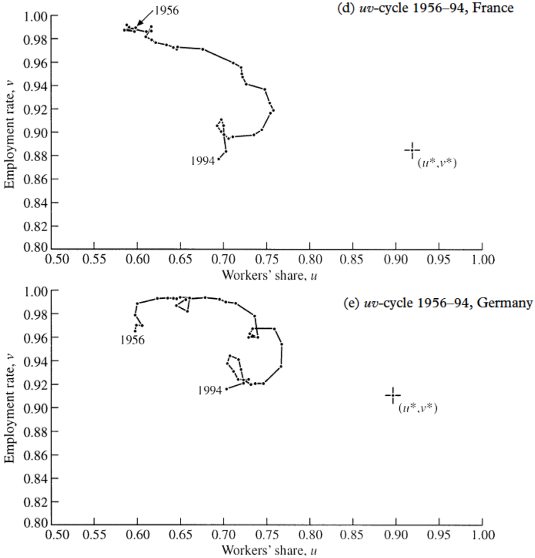
Example 1 (continued): Goodwin model



Goodwin Model - Extensions, structural instability, and empirical tests

- Desai 1972: Inflation leads to a stable equilibrium.
- Ploeg 1985: CES production function leads to stable equilibrium.
- Goodwin 1991: Pro-cyclical productivity growth leads to explosive oscillations.
- Solow 1990: US post-war data shows three sub-cycles with a "bare hint of a single large clockwise sweep" in the (ω, λ) plot.
- Harvie 2000: Data from other OECD confirms the same qualitative features and shows unsatisfactory quantitative estimations.

Testing Goodwin on OECD countries



Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

where $\kappa(\cdot)$ is $C^1(-\infty, \infty)$ satisfying

$$\kappa'(\pi) > 0 \text{ on } (-\infty, \infty)$$

$$\lim_{\pi \to -\infty} \kappa(\pi) = \kappa_0 < \nu(\alpha + \beta + \delta) < \lim_{\pi \to +\infty} \kappa(\pi)$$

$$\lim_{\pi \to -\infty} \pi^2 \kappa'(\pi) = 0.$$

Accordingly, total output evolves as

$$\frac{\dot{Y}}{Y} = \frac{\kappa(1 - \omega - rd)}{\nu} - \delta := g(\omega, d)$$

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y$$

Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned} \tag{1}$$

Good equilibrium

- Define
$$\overline{\pi}_1 = \kappa^{-1}(\nu(\alpha+\beta+\delta))$$
- We verify that
$$\overline{\omega}_1 = 1 - \overline{\pi}_1 - r\frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_1}{\alpha+\beta}$$
$$\overline{\lambda}_1 = \Phi^{-1}(\alpha)$$
$$\overline{d}_1 = \frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_1}{\alpha+\beta}$$
is an equilibrium for (1) and satisfies the relation
$$1-\overline{\omega}_1 - r\overline{d}_1 = \overline{\pi}_1$$
- Moreover
$$g(\overline{\omega}_1,\overline{d}_1) = \frac{\kappa(1-\overline{\omega}_1 - r\overline{d}_1)}{\nu} - \delta = \alpha+\beta.$$

Keen model - Explosive debt

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-r/u)}{\nu} - \alpha - \beta - \delta\right] \qquad (2) \\
\dot{u} &= u\left[\frac{\kappa(1-\omega-r/u)}{\nu} - r - \delta\right] - u^2\left[\kappa(1-\omega-r/u) - (1-\omega)\right].
\end{aligned}$$

- We now see that $(0, 0, 0)$ is an equilibrium of (2) corresponding to the point

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2) = (0, 0, +\infty)$$

for the original system.

Keen model - Local stability

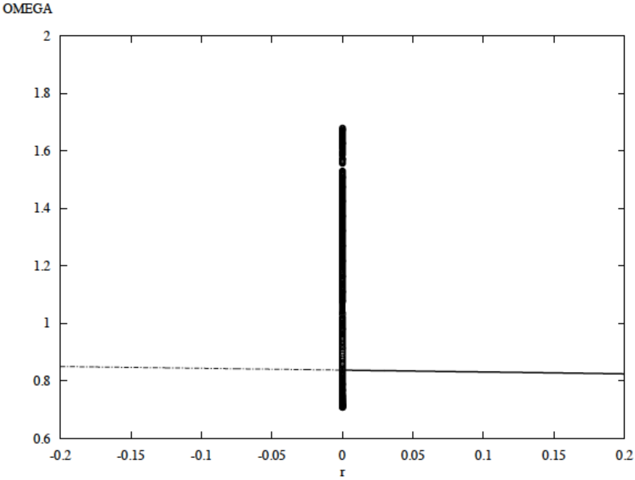
- Analyzing the Jacobian of (1) and (2) we obtain the following conclusions.
- The good equilibrium $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1)$ is stable if and only if

$$r\left[\frac{\kappa'(\overline{\pi}_1)}{\nu}(\overline{\pi}_1 - \kappa(\overline{\pi}_1) + \nu(\alpha+\beta)) - (\alpha+\beta)\right] > 0.$$

- The point $(0,0,0)$ is a stable equilibrium for (2) if and only if

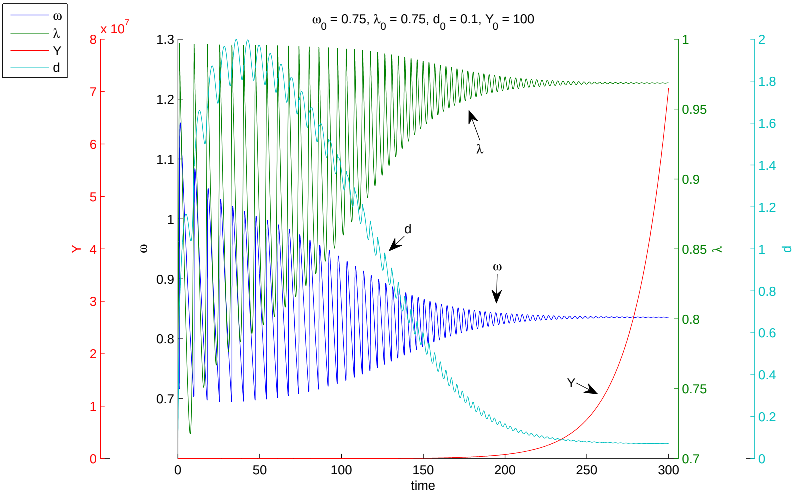
$$\frac{\kappa_0}{\nu} - \delta < r.$$

Interest rate bifurcation



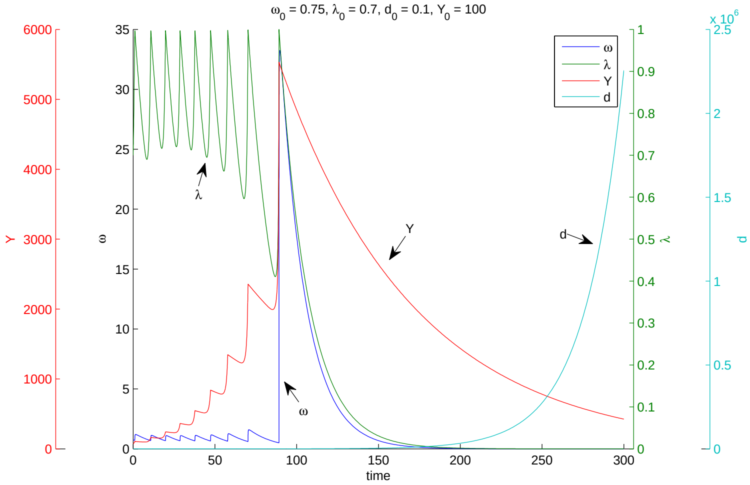
Example 2 : convergence to the good equilibrium in a Keen model

Example 2 (continued): convergence to the good equilibrium in a Keen model



Example 3: explosive debt in a Keen model

Example 3 (continued): explosive debt in a Keen model



Example 3 (continued): explosive debt in a Keen model

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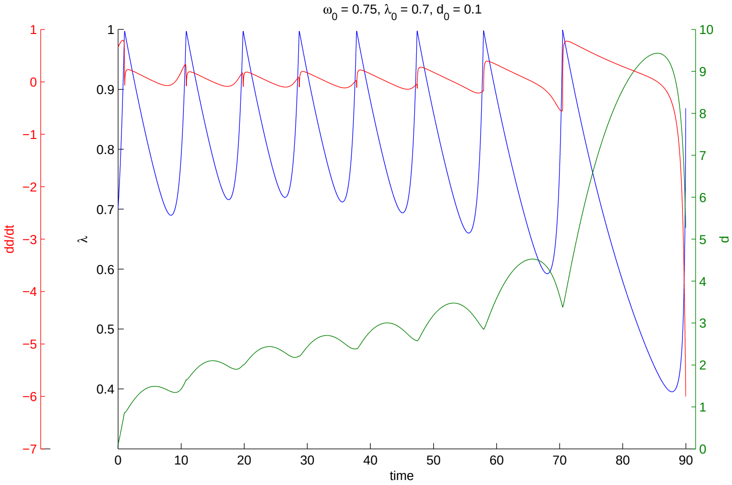
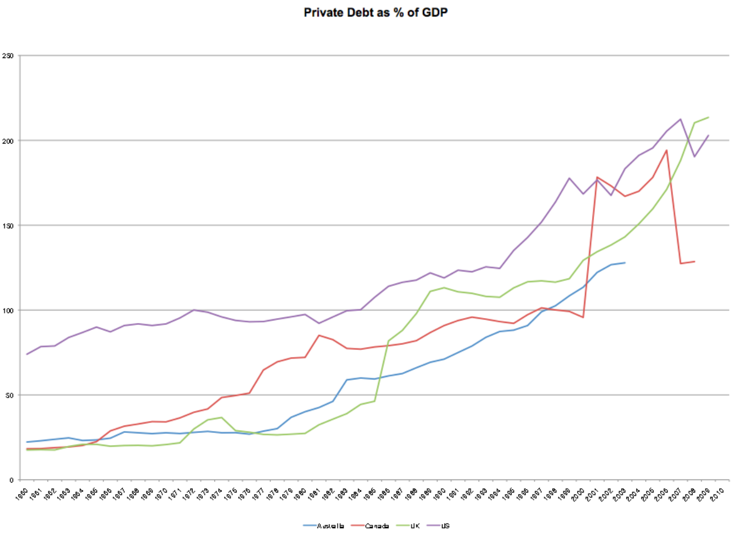
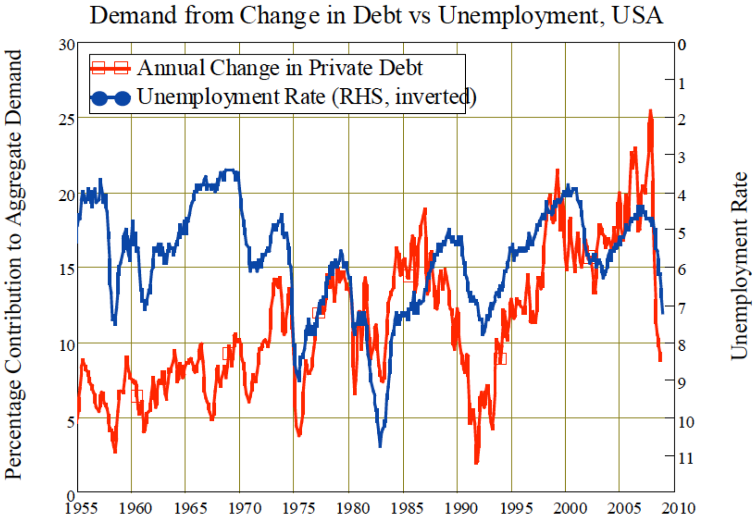


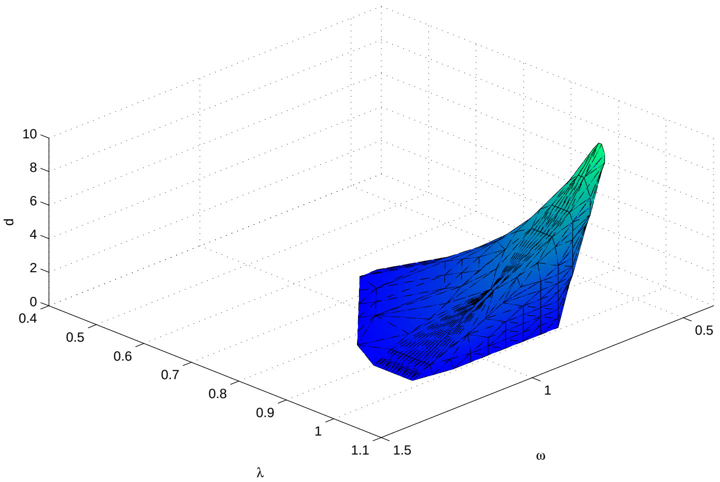
Data detour: debt



Data detour: debt and employment



Basin of convergence for Keen model



Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{D} = \kappa(1-\omega-rd)Y-(1-\omega-rd)Y+P$$
$$\dot{P} = \Psi(g(\omega,d)P$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g = \frac{\kappa(1-\omega-rd)}{\nu}-\delta.$$

Ponzi financing - Differential equations

With Ponzi financing the dynamical system becomes

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega) + p \\
\dot{p} &= p\left[\Psi\left(\frac{\kappa(1-\omega-rd)}{\nu} - \delta\right) - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right]
\end{aligned} \tag{3}$$

Ponzi financing - Equilibria and stability

- We find that $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1, 0)$ is a stable equilibrium iff

$$\Psi(\alpha+\beta) < \alpha+\beta.$$

- Introducing $u = 1/d$ we find that

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2, \overline{p}) = (0, 0, +\infty, 0)$$

is stable iff

$$\Psi(g_0) < g_0.$$

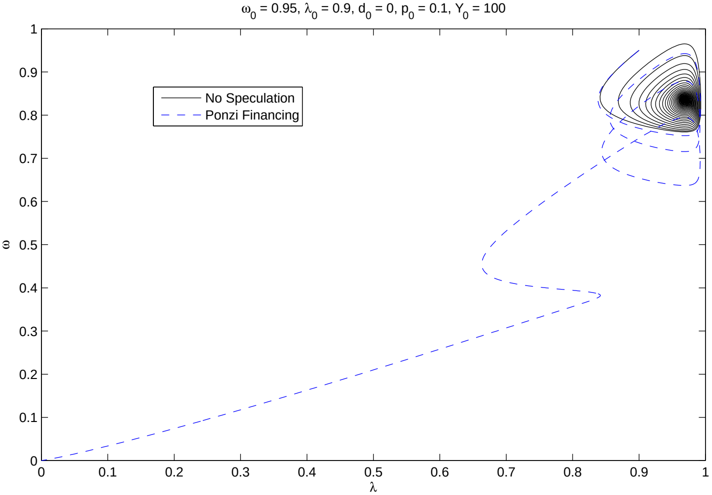
- Moreover, introducing , $x = 1/p$ and $v = p/d$ we find that

$$(\overline{\omega}_3, \overline{\lambda}_3, \overline{d}_3, \overline{p}) = (0, 0, +\infty, +\infty)$$

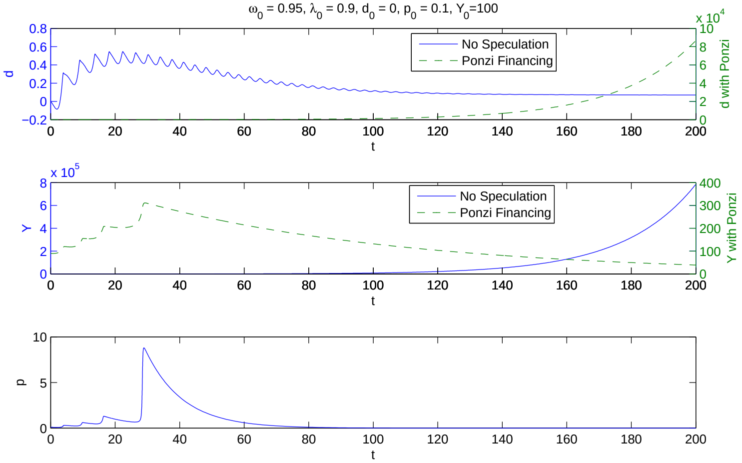
is stable iff

$$g_0 < \Psi(g_0) < r.$$

Example 4: effect of Ponzi financing



Example 4 (continued): effect of Ponzi financing



Introducing a government sector

- A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

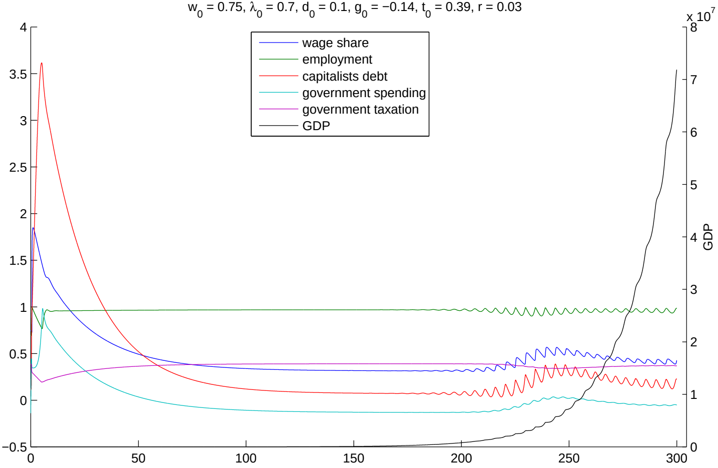
$$\dot{G} = \Gamma(\lambda)Y$$
$$\dot{T} = \Theta(\pi)Y$$

- Defining $g = G/Y$ and $t = T/Y$, the net profit share is now

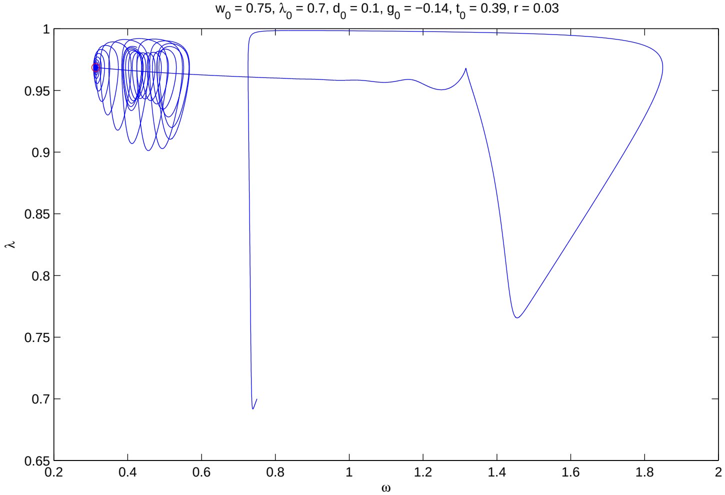
$$\pi = 1 - \omega - rd + g - t$$

- The new 5-dimensional system displays more local fluctuations, but no breakdown for the same initial conditions as before.

Example 5: stabilizing government



Example 5 (continued): stabilizing government



Stock prices

- Consider a stock price process of the form

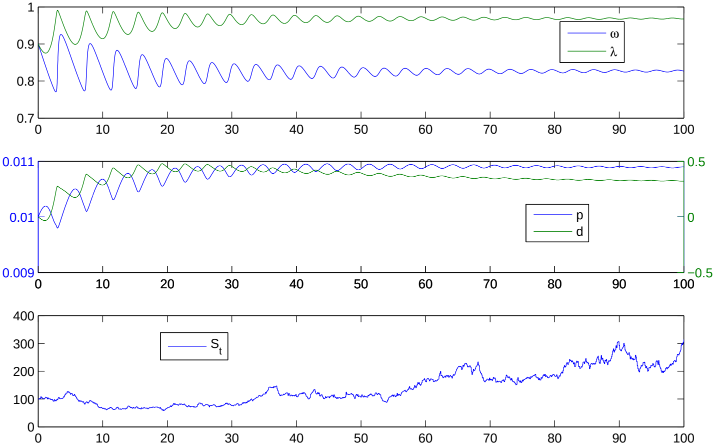
$$\frac{dS_t}{S_t} = r_b dt + \sigma dW_t + \gamma \mu_t dt - \gamma dN^{(\mu_t)}$$

  where N_t is a Cox process with stochastic intensity $\mu_t = M(p(t))$.

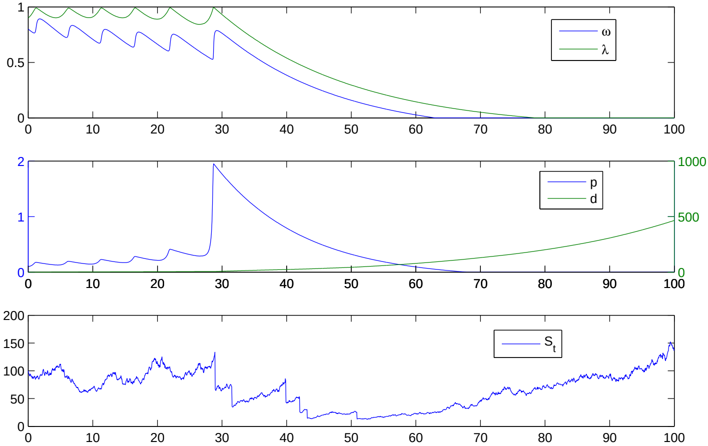
- The interest rate for private debt is modelled as $r_t = r_b + r_p(t)$ where

$$r_p(t) = \rho_1 (S_t + \rho_2)^{\rho_3}$$

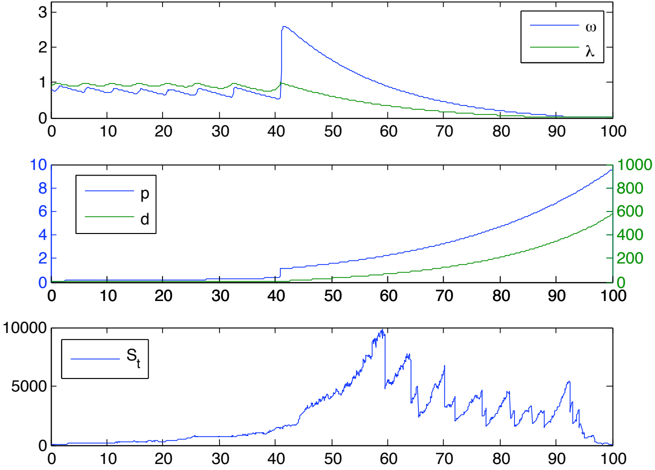
Example 6: stock prices, finite debt, finite speculation



Example 7: stock prices, explosive debt, zero speculation

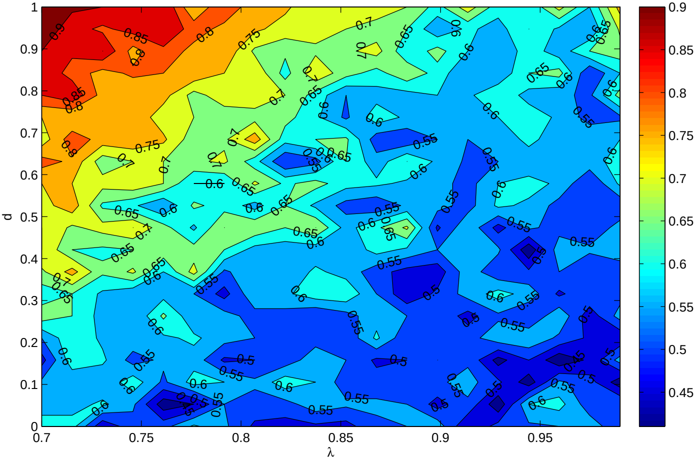


Example 8: stock prices, explosive debt, explosive speculation



Stability map

Stability map for $\omega_0 = 0.8$, $p_0 = 0.01$, $S_0 = 100$, $T = 500$, $dt = 0.005$, # of simulations = 100



Next steps

- Introduce delay in the investment function
- Characterize the equilibria with government sector
- Study stochastic model analytically
- Model prices for capital goods P_k and commodities P_c explicitly (Kaleckian mark-up theory, inflation, etc)
- Calibrate to macroeconomic time series.

Concluding thoughts

- Solow (1990): The true test of a simple model is whether it helps us to make sense of the world. Marx was, of course, dead wrong about this. We have changed the world in all sorts of ways, with mixed results; the point is to interpret it.
- Schumpeter (1939): Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them.