In search of the Minsky moment

M. R. Grasselli

Sharcnet Chair in Financial Mathematics
Mathematics and Statistics - McMaster University
Joint work with O. Ismail and B. Costa Lima

ORFE - Princeton University, April 4, 2011

Outline

Dynamic General Equilibrium views

- Seek to explain the aggregate economy using theories based on strong microeconomic foundations.
- Collective decisions of rational individuals over a range of variables for both present and future.
- All variables are assumed to be simultaneously in equilibrium.
- The only way the economy can be in disequilibrium at any point in time is through basing decisions on wrong information.
- Money is neutral in its effect on real variables.
- Largely ignore uncertainty by simply subtracting risk premia from all risky returns and treat them as risk-free.

Minsky's alternative interpretation of Keynes

- Neoclassical economics is based on barter paradigm: money is convenient to eliminate the double coincidence of wants.
- In a modern economy, firms make complex portfolios decisions: which assets to hold and how to fund them.
- Financial institutions determine the way funds are available for ownership of capital and production.
- Uncertainty in valuation of cash flows (assets) and credit risk (liabilities) drive fluctuations in real demand and investment.
- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.

Minsky's Financial Instability Hypothesis

- Start when the economy is doing well but firms and banks are conservative.
- Most projects succeed - "Existing debt is easily validated: it pays to lever".
- Revised valuation of cash flows, exponential growth in credit, investment and asset prices.
- Highly liquid, low-yielding financial instruments are devalued, rise in corresponding interest rate.
- Beginning of "euphoric economy": increased debt to equity ratios, development of Ponzi financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded.
- Euphoria becomes a panic.
- "Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing".

Rational bubbles: definition

- Consider a representative agent solving

$$\sup_c E_t \left[\sum_{j=1}^{\infty} \beta^{j-t} u(c_j) \right]$$

for exogenously given (e_t, d_t).

- Denoting $q_t = u'(e_t + d_t)p_t$, the FOC for optimality give

$$q_t - \beta E_t [q_{t+1}] = \beta E_t \left[d_{t+1} u'(e_{t+1} + d_{t+1}) \right]$$

- The general solution is of the form $q_t = F_t + B_t$ where

$$F_t = \sum_{j=1}^{\infty} \beta^j E_t \left[d_{t+j} u'(e_{t+j} + d_{t+j}) \right]$$

is the fundamental price and B_t is a bubble term satisfying

$$E_t[B_{t+1}] = \beta^{-1} B_t \tag{1}$$

Consequences

- $B_t \geq 0$ for all t.
- Any nonzero rational bubble must start with $B_0 > 0$.
- If $T < \infty$, $B_t = 0$ for all $0 \leq t \leq T$, and this result is robust with respect to diverse information (Tirole 1982).
- If $T = \infty$, bubbles can exit in a myopic rational expectations equilibrium.
- Rational bubbles cannot exist in a fully dynamic REE with finitely many infinitely lived agents.
- They can exit in an overlapping generations models provided $0 < \bar{r} < g$, where $\bar{r}$ is the asymptotic real interest rate and g is the rate of growth of the economy (Tirole 1985).

The Efficient Markets Hypothesis

- Denote $R_{t+1} = \frac{p_{t+1} - p_t + d_{t+1}}{p_{t+1}}$.
- As we have seen, a first-order rational expectations condition for risk-neutral agents leads to

$$E_t[R_{t+1}] = 1 + r. \tag{2}$$

- Solving this recursively leads to

$$p_t = \sum_{j=1}^{\infty} \frac{1}{(1+r)^j} E_t[d_{t+j}], \tag{3}$$

  plus a possible rational bubble term satisfying $E_t[B_{t+1}] = (1+r)B_t$.
- Either (2) or (3) can be taken as an EMH.
- Statistical tests on actual returns indicate that they are not *very* forecastable, leading to the conclusion that the EMH cannot be rejected.

Alternative models (Shiller, 1984)

- Consider a model where sophisticated investors have a demand function (portion of shares) of the form

$$Q_t^i = \frac{E_t[R_{t+1}] - \alpha}{\phi}. \tag{4}$$

- In addition, suppose there are noise traders who react to fads Y_t through a demand function $Q_t^n = Y_t/p_t$.
- In equilibrium we have $Q_t + \frac{Y_t}{p_t} = 1$.
- Inserting this into (4) and solving recursively leads to

$$p_t = \sum_{j=1}^{\infty} \frac{E_t[d_{t+j}] + \phi E_t[Y_{t-1+j}]}{(1+\alpha+\phi)^j}. \tag{5}$$

- This is also consistent with prices being not very forecastable.

Other sources of inefficiencies

- Noise trader risk (DeLong, Shleifer, Summers and Waldmann 1990): prices deviate from fundamental value because of uncertainty created by noise traders, who can in some cases earn higher expected returns than sophisticated investors.
- Limits of arbitrage (Shleifer and Vishny 1997): performance based arbitrage lead to fund managers leaving the market exactly when they are needed to restore fundamental value.
- No short-sales and diverse beliefs (Miller 1977, Harrison and Kreps 1978): pessimists sit on sidelines and optimists overbid leading to prices higher than fundamentals.
- Overconfidence (Scheinkman and Xiong 2003): mean reverting confidence levels lead to prices that contain an option to re-sell the asset at a later time.

Financial Intermediation (Allen and Gale, 2000)

- Risk-neutral investors with no wealth and banks with $B > 0$ funds to lend at rate r trading at $t = 1, 2$.
- Safe asset (s) with return $(1 + r)$ and a risky asset (R) with price at $t = 2$ given by a random variable p_2 with density $h(p_2)$ on $[0, p_2^{max}]$ and mean $\overline{p_2}$.
- The equilibrium price in the presence of banks is then

$$p_1 = \frac{1}{1+r}\left[\frac{\int_{(1+r)p_1}^{p_2^{max}} p_2 h(p_2) dp_2 - c'(1)}{\text{Prob}[p_2 \geq (1+r)p_1]}\right]. \qquad (6)$$

- Define the fundamental value as the price that an investor would pay if he had to use his own money $B > 0$.
- This leads to

$$p_1^F = \frac{\overline{p_2} - c'(1)}{1+r}. \qquad (7)$$

- It can shown that $p_1 \geq p_1^F$.

Modelling banks: liquidity preferences

- An asset is illiquid if its liquidation value at an earlier time is less than the present value of its future payoff.
- For example, an asset can pay $1 \leq r_1 \leq r_2$ at dates $T = 0, 1, 2$.
- Let $(r_1 = 1, r_2 = R)$ be an illiquid asset and $(r_1 > 1, r_2 < R)$ be a liquid one.
- At time $t = 0$, consumers don't know in which future date they will consume.
- The expected utility for consumers is

$$pU(r_1) + (1-p)U(r_2),$$

  where p is the proportion of early consumers.
- Sufficiently risk-averse consumers prefer the liquid asset.
- A similar story holds for entrepreneurs.

The Diamond and Dybvig (1983) model

- Banks borrow short and lend long.
- Suppose a bank offers a liquid asset ($r_1 = 1.28, r_2 = 1.813$) to 100 depositors each with \$1 at $t = 0$.
- In addition, the bank can invest in an illiquid asset ($r_1 = 1, r_2 = 2$).
- If $w = 1/4$, the bank needs to pay $25 \times 1.28 = 32$ at $t = 1$.
- At $t = 2$ the remaining depositors receive $\frac{68 \times 2}{75} = 1.813$ and the bank is solvent.
- This is a Nash equilibrium if *all* depositors expect only 25 to withdraw at $t = 1$.
- *But* liquidity preferences are unverifiable private information.
- Another Nash equilibirum consisting of *all* depositors forecasting that everyone will withdraw at $t = 1$.

Our model - the summarized story

- Society
- Liquidity Preference
- Searching for partners
- Learning and Predicting
- Bank birth
- Interbank Links
- Contagion

Society

- We have a society of individuals investing at the beginning of each period ($t = 0$).
- For each individual i, an initial preference is drawn from a continuous uniform random variable U_i: the investor is deemed to have short term liquidity preferences if $U_i < 0.5$ and long term liquidity preferences otherwise.
- There is a shock to their preferences at the middle of the period ($t = 1$).
- If the shock is big enough the individual would have wished he made his investment differently.
- At time $t = 1$, $W_i = \left| \frac{U_i + (-1)^{ran_i} \epsilon_i}{2} \right|$
- If $W_i < 0.5$ the investor wants to become a short term investor, otherwise he wants to be long term investor
- Because of anticipated shocks, individuals explore the society searching to partners to exchange investments.

Searching for partners

- We impose some constrains on the individual capacity to go around and seek other individuals to trade.
- This reflects the inherited limited capability of information gathering and environment knowledge of individual agents.
- We use a combination of von Neumann and Moore neighborhoods:

5	1	6
2	X	3
7	4	8

To join or not to join a bank

- Assume a bank offers a fixed contract promising a payment of $c_1 > 1$ at $t = 1$ for each unit (dollar) deposited and $1 < c_2 < R$ for $t = 2$ under the assumption there is no bank run.
- Then agents will join the bank if they have:
  1. short term preferences and expect not to change preferences in the next period
  2. short term preferences, expect to change preference and not find a partner to trade
  3. long term preferences and expects to change preference
- Agents will not join the bank if they have:
  1. short term preferences, expect to change and believes he can find a partner
  2. long term preferences and are confident they will not change

Bank birth

- We follow the work of Howitt and Clower (1999,2007) on the emergence of economic organizations
- With probability $0 < h < 1$ an agent will have the 'idea of entrepreneurship'
- Market search for an opportunity to establish a bank
- Establish a bank if he can find x and y such that $x + y \leq 1$ and

$$y = c_1 W_i$$
$$Rx = c_2(1 - W_i)$$

- Individuals become aware of bank existence only if the bank lies in their neighbourhood.
- In addition we give the bank the reach of its new members.

Experiment: bank formation

Experiment (continued): established banks

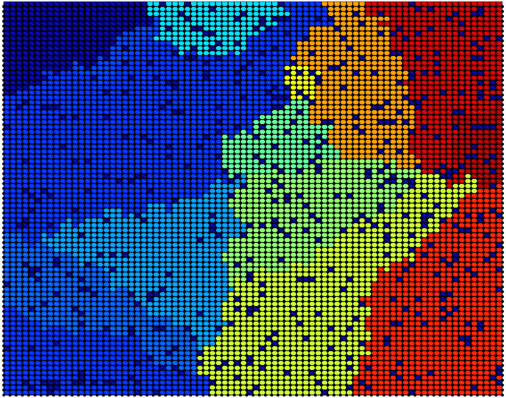
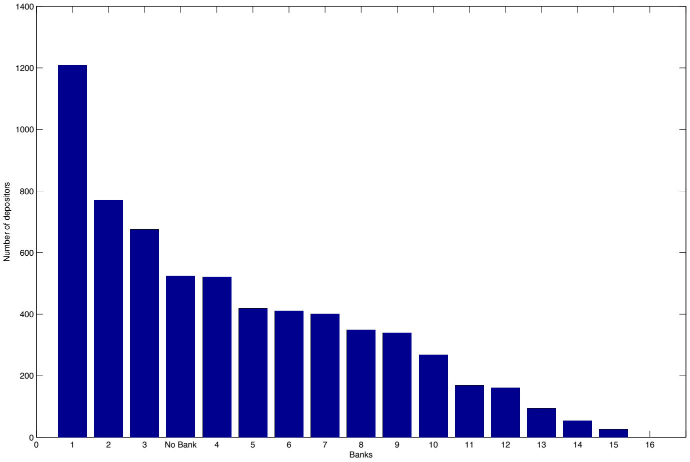


Figure: Banks at T=100 with $h = 0.9$, $c_1 = 1.1$, $c_2 = 1.5$ and $R = 2$.

Experiment (continued): number of depositors



Next steps

- Need to incorporate bank run
- Individuals moving between banks
- Banks form a new kind of agents that can in turn trade with each other to distribute the risk of asymmetric liquidity shocks a la Allen and Gale (2000):

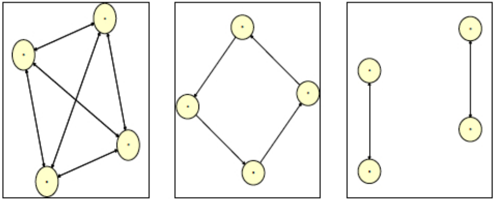


Figure: Networks, complete connected (left), incomplete connected (middle), incomplete disconnected (right)

Modelling Minsky: basic Goodwin Model

- Let $N = n_0 e^{\beta t}$ be the labour force, $a = a_0 e^{\alpha t}$ be its productivity and $\lambda = L/N$ be the employment rate.
- Define the total output $Y = aL$ and total capital as $K = \nu Y$.
- Assume that wages satisfy

$$\frac{dw}{dt} = F_w(\lambda) w,$$

where $F_w(\lambda)$ is a Phillips curve.
- Let the wages share of total output be ω and profit share be $\pi = 1 - \omega$.
- Suppose further that the rate of new investment is given by

$$I = \frac{dK}{dt} = (1 - \omega) Y - \gamma K$$

Differential Equations

- It is easy to deduce that this leads to

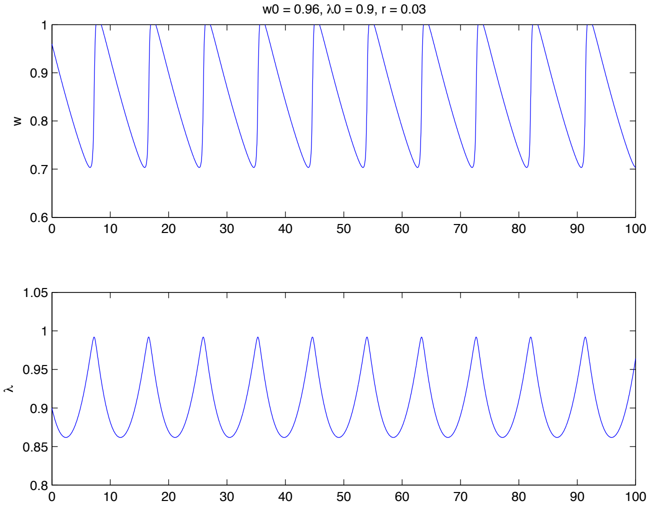
$$\frac{d\omega}{dt} = \omega(F_w(\lambda) - \alpha) \tag{8}$$

$$\frac{d\lambda}{dt} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \gamma - \beta\right) \tag{9}$$

- This system is globally stable and leads to endogenous cycles of employment.

Example 1: basic Goodwin model

Example 1 (continued): basic Goodwin model



Keen's extended model

- Consider the same model as before, but with a nonlinear investment function $I_g = \kappa(\pi_n/\nu)$ of the net profit share:

$$\pi_n = 1 - \omega - rd,$$

where $d = D/Y$ and the absolute debt level D evolves according to

$$\frac{dD}{dt} = I_g - \pi_n = rD + \kappa(\pi_n/\nu) - (1 - \omega).$$

- We then find that

$$\frac{1}{Y}\frac{dY}{dt} = F_Y(\omega, d), \tag{10}$$

where the growth rate taking into account the banking sector is now given by

$$F_Y(\omega, d) = \frac{\kappa\left(\frac{1-\omega-rd}{\nu}\right)}{\nu} - \gamma. \tag{11}$$

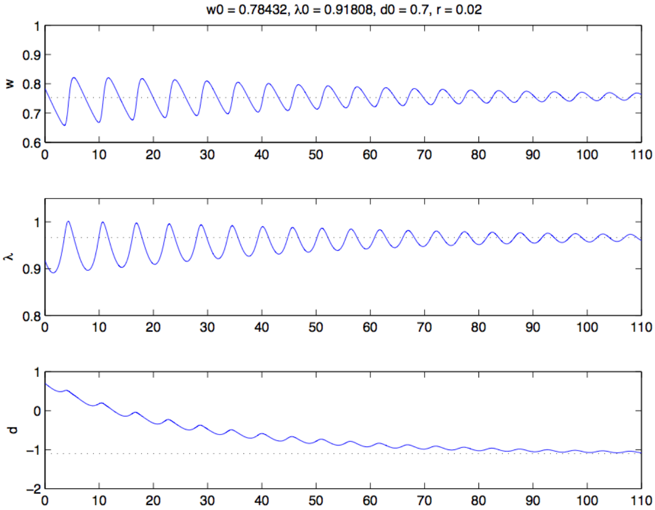
Differential Equations

- The corresponding dynamical systems now reads

$$\frac{d\omega}{dt} = \omega(F_w(\lambda) - \alpha)$$
$$\frac{d\lambda}{dt} = \lambda\left(F_Y(\omega, d) - \alpha - \beta\right)$$
$$\frac{dd}{dt} = d[r - F_Y(\omega, d)] + \nu[F_Y(\omega, d) + \gamma] - (1 - \omega)$$

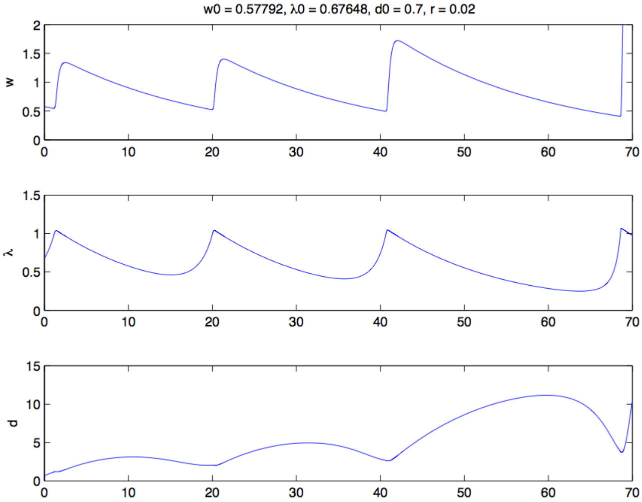
- This system is locally stable but globally unstable.

Example 2: convergent Goodwin model with banks

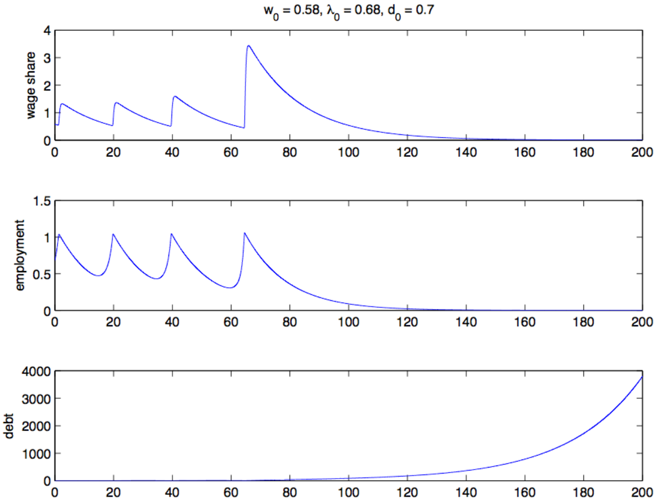


Example 2 (continued): convergent Goodwin model with banks

Example 3: divergent Goodwin model with banks

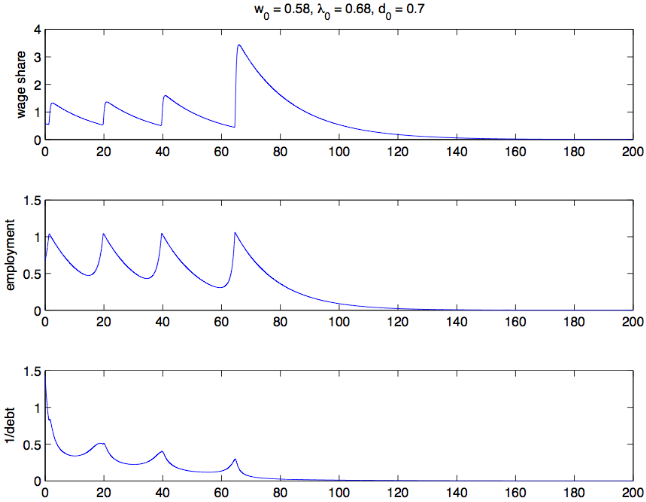


Example 3 (continued): divergent Goodwin model with banks



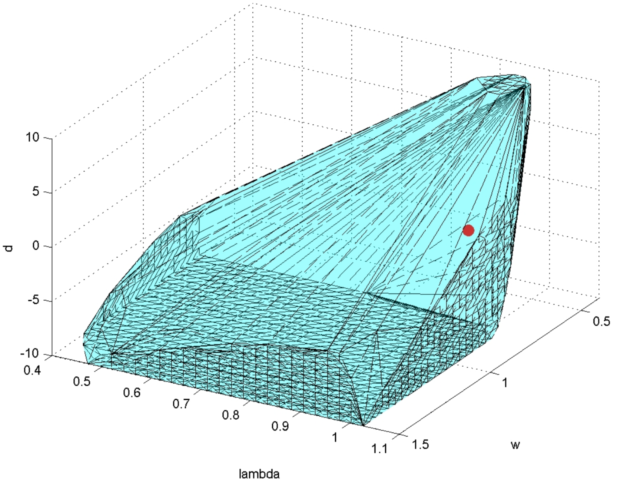
Example 3 (continued): divergent Goodwin model with banks

Example 3 (continued): divergent Goodwin model with banks



Example 3 (continued): divergent Goodwin model with banks

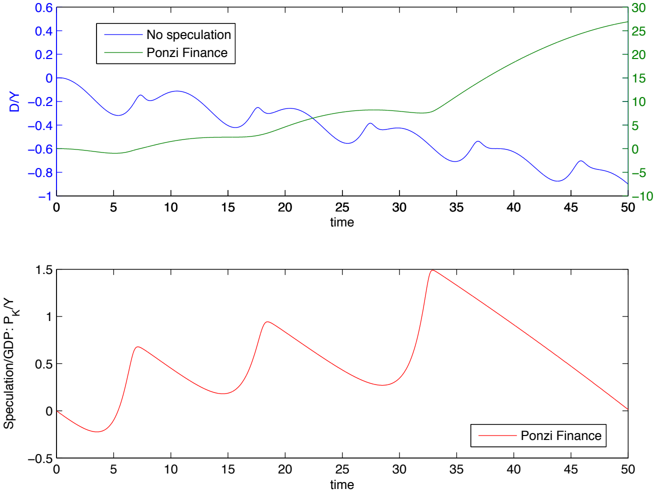
Basin of convergence for Goodwin model with banks



Ponzi financing

- To introduce the destabilizing effect of purely speculative investment consider a modified version of the previous model with
$$\frac{dD}{dt} = I_g - \pi_n + P_k,$$
where
$$\frac{dP_k}{dt} = F_p(F_Y)$$
- Here $F_p(\cdot)$ is a increasing nonlinear function of the growth rate of economic output F_Y.

Effect of Ponzi financing



Next steps

- Add government (regulatory) sector.
- Model asset prices P_k explicitly.
- Introduce noise (stochastic interest rates, risk premium, etc)
- Thanks !