# Pricing Sovereign Bonds

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**Abstract**

We begin to introduce the Socereign Bonds by reviewing the nature of sovereign credit risk (rating and recovery). Then we construct a model for pricing sovereign debt that accounts for the risks of both default and restructuring, and allows for compensation for illiquidity.We consider the determinants of the Russian debt as an example.

## 1 introduction

Among all forms of debt, sovereign bonds have long enjoyed a special status.

**Definition 1** *A* ***sovereign bond*** *is a bond issued by a national government as opposed to a municipal bond which is issued by a subdivision of a national government and as opposed to a corporate bond which is issued by a corporation.*

A holder of sovereign debt may not have recourse to a bankruptcy code in the event of default.Moreover, a sovereign default is largely a political decision,influenced by such macroeconomic factors as the balance of payment,central bank reserves, and so on. Governments trade off the cost of making debt payments against reputation costs(Eaton and Gersovitz (1981)),the costs of having assets abroad seized, and the costs of having international trade impeded (Bulow and Rogoff (1989b) and Gibson and Sundaresan (1999)). A sovereign rarely makes an outright default. Rather, it may force a restructuring or renegotiation of its debt. Indeed, the same bond may be repeatedly renegotiated (Bulow and Rogoff (1989a)).A sovereign also trades off the costs of default (or forced restructuring) of internal versus external debt. As we shall see, this trade-off has interesting implications for the pricing of different classes of sovereign debt.An example of Russian debt will be given later.We are not aware of any theoretical papers that address this issue. Indeed, most papers on sovereign debt address only external debt. Kremer and Mehta (2000) study a model in which a government is more inclined to default if a large proportion of its debt is held by foreigners.

# 2 Credit Risk in Sovereign Bonds

## 2.1 The Basic Knowledge of Sovereign Bonds

Sovereign Risk is rarely one dimensional. Among the types of credit events that holders of sovereign debt might experience are:

- *Default or repudiation* :The sovereign announces that it will stop making payments on its debt.

  **Definition 2** ***Default*** *is sometimes modeled as the event in which, at maturity, there are insufficient assets to pay down the debt,or the event that the debtors cash-flows or asset-liability ratio falls below some cut-off level for the first time.*

- *Restructuring or renegotiation* :The sovereign and the lenders "agree" to reduce (or postpone) the remaining payments.
- A *regime switch* , such as a change of government or the default of another sovereign bond that changes the perceived risk of future defaults.

Sovereign debt shows a basic difference in comparison to regular corporate debt, which is the impossibility of seizing any asset located inside the issuer country. Therefore, most of the countrys net worth (except what is located outside) is worthless for the lender. There is no way to guarantee that it will pay its debt, even if it has enough money. Moreover, during the problem of enforcement, moral hazard and adverse selection are worse in sovereign than in corporate debt. There are two basic issues in sovereign debt: ability and willingness to pay.

The ability to pay, or solvency, is a cash-flow analysis of the countrys external account. It can have different degrees of sophistication and may include or not a dynamics for interest rate, external supply of funds, current account, etc. It also depends on the government effort to collect revenues and change them for hard currency to service their debt.An extreme version of this case is given by Simonsen (1985), who defines the solvent borrower as the one whose trade account present value is larger than the current debt value.

The willingness to pay is in fact a choice between the loss that comes from a default and the money saved on debt payment. This loss is proportional to the penalties that borrowers believe will be imposed by the lenders, and the latters willingness to lend again in the future. In fact, shortly after a default, countries re-enter the voluntary market, lenders shall limit the maximum amount available for sovereign borrowers to the present value of the possible penalties these countries can suffer in case of default.

Although willingness to pay poses the strongest limit on the debt size, the default has been associated with adverse economics shocks, generating what they call excusable default. So in practice, ability to pay has been the greatest factor to affect sovereign debt prices. Ability to pay is easier to quantify than willingness, and most pricing models are based on ability constraint.

There are three ways to evaluate the sovereign debt: the first a reduced form, where the debt price is explained by a set of external and internal economic variables without an theoretical connection. The second approach is to find a process for the default and based on this process, find the risk neutral price of the debt. The problem with this approach is obtained with some existing debt, therefore is useful only for arbitrage among different types of debt. Another approach, a more structural one, presents a relationship between fundamental variables and debt price.

In order to discuss the pricing models,we briefly introduce the nature of the credit risks of sovereign bonds.

## 2.2 The Nature of Sovereign Credit Risk

The foreign-currency-denominated sovereign bonds had a low default probability over the past two decades. For example, there had never been a restructuring or default on a sovereign Eurocurrency bond.However, when Pakistan launched an offer to exchange or restructure some its Eurobonds in November 1999, it was the first time that private investors faced a loss on Eurobonds. Meanwhile, the International Monetary Fund(IMF) didn't share in the losses with private investors. In this case, Pakistan in question were restructured ro an actual default event, given that investors viewed the threat of default as credible in the absence of a restructuring(International Monetary Fund,2001). subsequently, Ecuador defaulted on some of its Eurocurrency bonds, and these bonds were in default for about a year. This was the first default on bonds that were,at the time,widely traded on secondary markets.Now, let us turn on attention to the secondary market temporarily.There are two ways in which bonds can be traded in the secondary market of the Eurozone. The traditional way is through an organized exchange were trading has been fairly low. The second way is through the OTC market in which the main players are banks, most of them also participating in the primary auctions. The secondary market decides upon the trading environment.In particular, it determines the structure of payments and settlements and the trading facilities offered by brokers and market makers. Both sectors influence the price dynamics through supply and demand, where the primary sector acts as the ultimate provider of liquidity. It is therefore useful to give a description of the Eurozone government bond market based on these two sectors.

Now,let us back to the foreign-currency-denominated sovereign bonds.There are many types of foreign-currency-denominated sovereign debts,for example:

- debts governed by the domestic law of the issuer (e.g.,Russian Ministry of Finance bonds denominated in U.S dollars)
- Eurocurrency bonds,
- debts,such as Brady bonds, that arise from prior restructurings of bank loans or bonds(The major difference between borrowing by bond and loan is that under bond finance a default is necessary to start a renegotiation, while in the case of loan finance the default can be avoided with the help of the banks until some rescheduling is reached.)

The response by debtholders to a default or proposed restructuring may be very different in each of these cases. Further, as in the Russian case, which we will discuss more later, portions of the outstanding debt may have been issued under different political regimes. In this situation, the current regime may feel a stronger obligation to make contractual payments on the debt issued during its own regime.

Most Eurocurrency bonds are governed by the laws of either the United Kingdom (approximately 25 % of issues) or the United States (approximately 75 % of issues) Specifically, under U.K. law, in order to modify the payment terms of a bond, consent by a quorum of 75 % is needed. In contrast,under U.S law, a quorum of 100 % is needed to initiate a restructuring of bond issues in a credit event. Thus, U.K law is more likely to encourage collective action on the part of bondholders and perhaps lead to less time between a credit event and the consumation of a restructuring.

There are at least 3 key players on the creditor side:

- The London Club, representing more than 600 Western commercial lenders that own debts that have been restructured by sovereign issuers;
- The Paris Club, consisting of Western governments that have lent money to developing nations and have agreed not to accept restructuring terms less favorable than those offered to the London Club,
- Private Lenders, which may form a relatively heterogeneous group.

In assessing the creditworthiness of a sovereign,some have focused on its debt-service capacity. That is, the probability of default is linked to the sustainability of external debt and to problems associated with short-term illiquidity or long-run solvency.Among the macroeconomic variables that are likely to affect a sovereign's ability to service its debt(and the expected direction of the effect)are: current account to GDP(+),terms of trade(+),reserves to imports(+),external debt(-),income variability(-),export variability(-),and inflation(-).

Measurement of recovery is particularly challenging for sovereign bonds because so many of the bonds are restructured, often many times, and even in the absence of a formal declaration of default. For the Latin American defaults during the 1980s, the losses at default range from 10 to 50%, with an average of approximately 25%(Deutsche Bank Research,1999),imply relatively high recovery rates.

Within the sphere of Eurobonds, we would expect that the place of legal registration of the bonds might affect recovery.As noted previously,collective action by creditors to change the payment structure of debts is much easier under U.K. law than under U.S. law. Recovery may, on average, be correspondingly quicker and perhaps greater under U.K law. Interestingly, some countries have Eurobonds outstanding that are denominated in the same currency but governed by different legal systems. Turkey and the Philippines are relevant examples. A natural question, then, is whether market prices reflect the different probabilities of default and restructuring associated with the relative case of collective action. The analysis of Deutsche Bank Research(1999) suggests that prices do not reflect these differences. On the other hand, in a larger and more systematic study, Eichengreen and Mody(2000) find that such clauses lower borrowing costs for high-quality sovereign issues because of the benefits of easier restructuring and associated higher expected recoveries. On the other hand, for low-quality sovereign issuers, the costs of moral hazard and default risk associated with the relative ease of creditor-initiated restructuring seem to be associated with wider spreads.

# 3 The case of Russian Bonds

We proceed by discounting cash flows promised by a sovereign bond using a short-term discount rate adjusted for the risk-neutral mean loss rate of the various types of credit events discussed earlier. Additionally,we accommodate the possibility that bonds issued by the same sovereign, of exactly the same type but possibly of different maturities, may be priced in the market using different discount factors. Reasons for this may include:

- Bond covenants may not include cross-default clauses that would force, upon the default of one bond, the simultaneous default of other bonds of the same type, but of a different maturity. For various strategic reasons related to internal or external political or economic considerations, sovereign issuers may choose to default on, or to renegotiate, the terms of one bond (or one set of bonds), but not on others.

- Some sovereigns (such as Turkey and the Philippines) have issued Eurobonds governed by different legal systems (those of the United Kingdom and the United States) that have different collective-action clauses, and, hence, differ with respect to the ease of creditor-initiated restructurings.

- Portions of the outstanding debt may have been issued during different political regimes. In this situation, the current regime may feel, or be perceived to feel, a stronger obligation to make contractual payments on debt issued during its own regime.

- Finally, for these reasons and because of possible clientele trading patterns, high transaction costs, or asymmetrically informed traders (including government insiders), different bonds may have different illiquidity.

As an illustration of our valuation framework, we estimate a model of the term structure of credit spreads for bonds of the Russian Ministry of Finance (MinFin) over a sample period encompassing the default on domestic Russian GKO bonds in August 1998.For this purpose, our model is parameterized with the composite credit-and-liquidity spread of one of the Russian MinFins serving as a "benchmark". All other MinFins are allowed to have idiosyncratic components to their discount rates relative to this benchmark. Importantly, these idiosyncratic factors enter directly into the discount rates, rather than being treated as additive pricing or measurement errors, as has become common practice in the literature on dynamic term structure models. Initially, our model is estimated using data up to, but not including the 1998 Russian default.

The case of Russian bonds, and in particular the Russian MinFins that Duffie focus on in their empirical analysis, highlights the potential limitations of applying corporate-bond pricing models to sovereign debt. Table 1 lists five dollar-denominated MinFin bonds issued in 1993 as payment to Russian exporters for Vnesheconombank accounts that were frozen in 1991.In 1996, two additional MinFins were issued, and Russia issued its first Eurobond. Several more Eurobond issues followed in 1997 and 1998.

Based on past experiences with Soviet-era debt and the debts of other sovereigns, investors in these bonds would surely have recognized that outright default was not the only possible, or indeed even the most likely, credit event. More likely would be a restructuring, with possible additional restructurings prior to maturity. In fact, between April 1993 and August 1999, the

Paris Club of Western government creditors restructured its Soviet-era debts with Russia five times. Moreover, as part of any restructuring, there is the possibility of writedowns of principal.The Russian Federation defaulted on the principal payments at maturity of the MinFin 3 bonds that matured in May 1999 (essentially, a writedown at maturity).More recently, the restructuring of debt held by the Russian London Club of Western commercial lenders announced in early 2000 called for both writedowns and change of obligor status. In 1997, the London Club restructured its Soviet-era debt into two securities: six billion dollars of principal notes (Prins) and 20 billion dollars of interest-arrears notes (IANs).The subsequent restructuring announced in 2000 called for reductions in principal of 37.5 percent and 33 percent of the Prins and IANs, respectively, and a change in obligor status fromVnesheconombank to the Russian Federation through the issue of Eurobonds (InternationalMonetary Fund (2000a)).

Moreover,although MinFins are denominated in U.S. dollars, they are technically domestic debt under the jurisdiction of Russian law and have no cross default clauses. This gives the Ministry of Finance some discretion in deciding which creditors are most severely affected by a credit event (e.g., which bonds will default), as well as discretion over the nature of a credit event, including the effective recovery rate.

Another feature of Russian MinFins is the importance of "political" risk.The Ministry of Finance has drawn a clear distinction between their commitments to fulfill the principal obligations of "Soviet-era" versus "post-Soviet-era" debt. MinFins 3 through 5, as indicated in Table 1, were issued in 1993, and hence are technically "post-Soviet-era" debt, but were issued to address "Soviet-era" problems. There has been no explicit commitment by Russia to repay their principals at maturity. In fact, Russia defaulted on the principal obligation of the MinFin 3 bonds in 1999. In contrast, the Russian Ministry of Finance has publicly committed to fulfill all of its payment obligations on the MinFin 6 and MinFin 7, as well as on the Eurobonds.

The effects of the Russian default on GKOs in August, 1998 on the prices of MinFins are displayed in Figure 1,

which shows the prices of five MinFin bonds from January 2, 1998, to November 17, 2000, relative to their prices on July 31, 1998 (just prior to default). Note, first of all, that MinFin prices fell by approximately 80 percent during the week of the Russian default, suggesting that market participants were indeed surprised by this credit event right up to the time of the event itself. This is roughly consistent with default arrival at an inaccessible stopping time, as in our intensity-based modeling approach.

Second, the general patterns of relative prices around the time of default suggest that a fractional recovery of market value is not an unreasonable assumption relative, say, to a typical alternative assumption that bondholders would recover an exogenous fraction of the bonds face value.Third, the fact that the price of the MinFin 3 did not approach its face value just prior to maturity suggests that many investors believed, prior to May 1999, that they would not receive 100 percent of face value at maturity of these MinFins. Essentially, this bond had already gone into default in August, 1998, although the precise terms of default were not known at that time. Likewise, we could think of the other MinFins as being in default after August 1998. These bonds have recently recovered substantially, however, which we interpret as a sign of investors being more optimistic about the impact of the (implicit) default of August 1998 on the eventual payments at maturity. This optimism was also reflected in the index of Russian stock prices shown in Figure 1.

Fourth, the MinFins recovered about 20 to 30 percent of their previous market values after the credit event of August 1998, whereas the Eurobonds (not shown) recovered around 30 to 40 percent of their market values. Additionally, Figure 1 shows that the MinFin 6 and MinFin 7 bonds have recovered larger fractions of their predefault values than have theMinFin 4 and 5 bonds. Hence, since the August 1998 default, investors may have been viewing Eurobonds and post-Soviet MinFins as being of higher quality than Soviet-eraMinFins.We shall later examine whether this viewpoint was reflected in predefault prices.

# 4 A Pricing Model of Sovereign Debt

Initially, we characterize the price,$P_t$,at any time $t$ before default of a sovereign security that promises to pay a single, possibly random, amount $Z$ at some time$T > t$.We fix a probability space and an increasing family $\{\mathcal{F}_t : t > 0\}$ of information set defining the resolution of information over time, with$Z \in \mathcal{F}_T$.We take as given a short-rate process $r$ and an equivalent martingale measure $Q$.That is, a security paying some amount $\tilde{Z}$ at some time (possibly a stopping time) $S$ has a market value at any time $t$ before $S$ of $E_t^Q[e^{\int_t^s -r(u)du}\tilde{Z}]$, where $E_t^Q$

denote $\mathcal{F}_t$-conditional expectation with respect to $Q$. Uniqueness of the risk-neutral measure $Q$ is not to be expected in our setting.

For the case of a single credit event (default), Duffie and Singleton (1999) show that for pricing purposes we can treat the cash flow $X$ as default free and allow for default risk by replacing the discount rate $r(t)$ with the "default-adjusted" discount rate $r(t)+h(t)L(t)$, where h is the risk-neutral intensity of default and L is risk-neutral expected fractional loss in market value in the event of default. In modeling sovereign spreads, we extend this model to one that accommodates many types of credit and liquidity events, in particular the "default" or "regime switch" types enumerated in section 2.1.This yields a pricing formula similar to that derived (in a less general setting) by Schönbucher (1998), who considers multiple defaults.

To accommodate multiple types of credit events, we let$N_t$ be the number of credit events of any type that have happened by time $t$, where $N$ is a counting process with risk-neutral intensity process $\lambda^Q$.We assume that, at the $n$th credit event, the promised payment of the security is lowered to a fraction,$Y_n$ of its pre-credit-event value. For example, a sovereign issuer may simply unilaterally announce at the first credit event that the principal that it recognizes as an obligation is reduced by 50 percent, in which case $Y_1 = 0.5$. This leaves open the possibility of a further repudiation at a later date. Also, at the $n$th credit event, investors may receive, or prospectively value, cash flows paid by the issuer in lieu of the repudiated portion of the debt, with a current market value that is a fraction $W_n$ of the pre-credit-event market value of the obligation.We assume that $\lambda^Q$, $Y_n$, and $W_n$ are exogenously given.The total effective fractional loss in market value caused by the $n$th credit event is thus $1 - Y_n - W_n$. At time t,the risk-neutral expected fractional loss associated with the next credit event, were it to occur immediately, is denoted $L_t^Q$. Hence, the process describing the risk-neutral mean fractional loss rate (due to credit events) is $\lambda^Q L^Q$.

We also allow liquidity to affect pricing. As in Duffie and Singleton (1999), we make the simplistic assumption that illiquidity of the security translates into a fractional cost rate of $l$, where $l$ is a predictable process. Hence, the total mean loss rate of the security due to credit events and illiquidity is $\lambda^Q L^Q + l$.If the repudiated fraction $Y_n$ is observable at the event time $\tau_n$,then, at time $t$, the security is worth the fraction$\Pi_{n \leq N(t)} Y_n$ of an otherwise identical security that has not yet experienced credit events.This gives the intuition for the following pricing formula.

**Theorem 4.1** *Let $R = r + s$, where*

$$s = \lambda^Q L^Q + l \tag{1}$$

*Suppose that $V_t = E_t^Q[e^{\int_t^T -r(u)du} Z]$ is well defined, and that (almost surely), the process $V$ does not jump at a credit event time. If the repudiated fraction $Y_n$ is observable at the event time $\tau_n$, then,*

$$P_t = E_t^Q(e^{\int_t^T -r(u)du} Z)\Pi_{n \le N(t)} Y_n \tag{2}$$

Proof: The discounted gain process is

$$G_t = -\int_0^t e^{-\int_0^v r_u du l_v} P_v dv + -\int_0^t e^{-\int_0^v r_u du} W_{N_v} P_{v-} dN_v + -\int_0^t e^{-\int_0^v r_u du} P_t \tag{3}$$

The first term is the discounted costs of illiquidity. The second term is the discounted cash flows generated at the times of credit events, which are the times at which $N$ jumps by one. The third term is the discounted price.With the proposed price process $P$ stated in the theorem, the final price is$P_T = Z\Pi_{n \le N(t)} Y_n$,as required.After an application of Itô's Formula,as in Duffie and Singleton (1999), the discounted gain process is a Q-Martingale, using the fact that $N_t - \int_0^t \lambda_u du$ is a Q-Martingale. There properties of $P$ and $G$ are necessary, from the definition of $Q$ as an equivalent martingale measure, for $P$ to be the price process.There is a unique price process with this property, as explained in Duffie and Singleton (1999).

We refer to the process s in equation (1) as the short (or instantaneous) spread. The condition that $V$ does not jump at a credit event time is automatically satisfied in our model of Russian bond prices because $V_t$ depends on the current level of a diffusion process.

If the repudiated fraction $Y_n$ is observable at the event time $\tau_n$, then, at time $t$, the security is worth the fraction $Y_1 Y_2 ... Y_{N(t)}$ of an otherwise identical security that has not yet experienced credit events.Even without formal econometric analysis, the behavior of the prices ofMinFin 3 shown in Figure 1 suggests that allowing for the risk of writedowns that are instigated by the default on GKOs is critical for pricing Russian bonds. As the MinFin 3 matured in 1999, its price approached about a third of its face value. In a purely intensity-based model of credit risk, however, because MinFin 3 was not in default, its price must approach face value at maturity.

Not only does the observed pattern call for a writedown, but it challenges the strong assumption of Theorem 4.1, that the fraction $Y_n$ of the notional that is recovered due to the nth credit event is observable at the associated credit event time $\tau_n$ An empirically more plausible view is that, as a consequence of the default on GKOs in August, 1998, bond prices reflected investorsexpected payment at maturity, and that investorsexpectations were subsequently revised upwards over time toward one-third of face value as market and economic conditions changed.

To formalize the idea of "expected payments at maturity",we allow for bond investors to learn more about the extent of the writedown after the time of the credit event itself, including the possibility that the final resolution of the impact of a restructuring is learned only at maturity. Although such learning substantially increases the difficulty of pricing sovereign bonds, we can extend our pricing relationship under the (unrealistically strong) assumption of *Independent Recovery*: Risk-neutrally, $\{Y_1, Y_2, ...\}$ are independent,and independent also of $\{Z, r, l, N, W_1, W_2, ...\}$.With this assumption, our pricing result extends as follows.

**Theorem 4.2** *Let $R = r + s$, where $s = \lambda^Q L^Q + l$ Suppose that $V_t = E_t^Q[e^{\int_t^T -r(u)du} Z]$ is well defined, and that (almost surely), the process $V$ does not jump at a credit event time.Under the assumption of Independent Recovery,*

$$P_t = E_t^Q(e^{\int_t^T -r(u)du} Z)\Pi_{n \leq N(t)} E_t^Q(Y_n) \tag{4}$$

The Independent Recovery assumption is strong because, for instance, as economic conditions in Russia improve, we would expect both that$E_t^Q(Y_n)$ is revised upward and that the credit-event arrival intensity h declines.The independence assumption does, however, allow for state-dependent writedowns $E_t^Q(Y_n)$ that substantially increase the flexibility of our model in capturing market price movements. In particular, subsequent to the first credit event, when at least one $E_t^Q(Y_n)$ term is present in equation (4),bond prices may jump up or down as investors revise their expectations about the consequences of past credit events for eventual receipts at maturity. Such jumps might occur at any time, including just prior to the maturity of the bond. A notable example of the type of jumpiness that one may capture with stochastic $E_t^Q(Y_n)$ is the rather saw-toothed pattern of the price of MinFin 3 in 1999 prior to its default at maturity (see Figure 1).We note that jumps in $E_t^Q(Y_n)$ are conceptually distinct from jumps in prices that might occur as a consequence of jumps in the short spread $s$, due say to a jump in the risk-neutral intensity of credit events.

# 5 conclusions

By introduced the sovereign bonds, we develop a model for pricing sovereign debt that captures the risks of several credit events, most importantly the risk of restructuring. The model is for estimated the Russian dollar-denominated sovereign bonds which you can find more technique part in "Modeling term structures of defaultable bonds" by Duffie and Singleton (1999).

**Table I**

## Contractual Characteristics of Russian Dollar-denominated MinFins and Eurobonds

| Issue | Issue date | Maturity | Coupon | Amount issued (billions of U.S.$) |
|---|---|---|---|---|
| MinFin 3 | 5/14/1993 | 5/14/1999 | 3.00 | 1.32 |
| MinFin 4 | 5/14/1993 | 5/14/2003 | 3.00 | 3.38 |
| MinFin 5 | 5/14/1993 | 5/14/2008 | 3.00 | 2.84 |
| MinFin 6 | 5/14/1996 | 5/14/2006 | 3.00 | 1.75 |
| MinFin 7 | 5/14/1996 | 5/14/2011 | 3.00 | 1.75 |
| Eurobond-01 | 11/27/1996 | 11/27/2001 | 9.25 | 2.40 |
| Eurobond-07 | 6/26/1997 | 6/26/2007 | 10.00 | 1.00 |

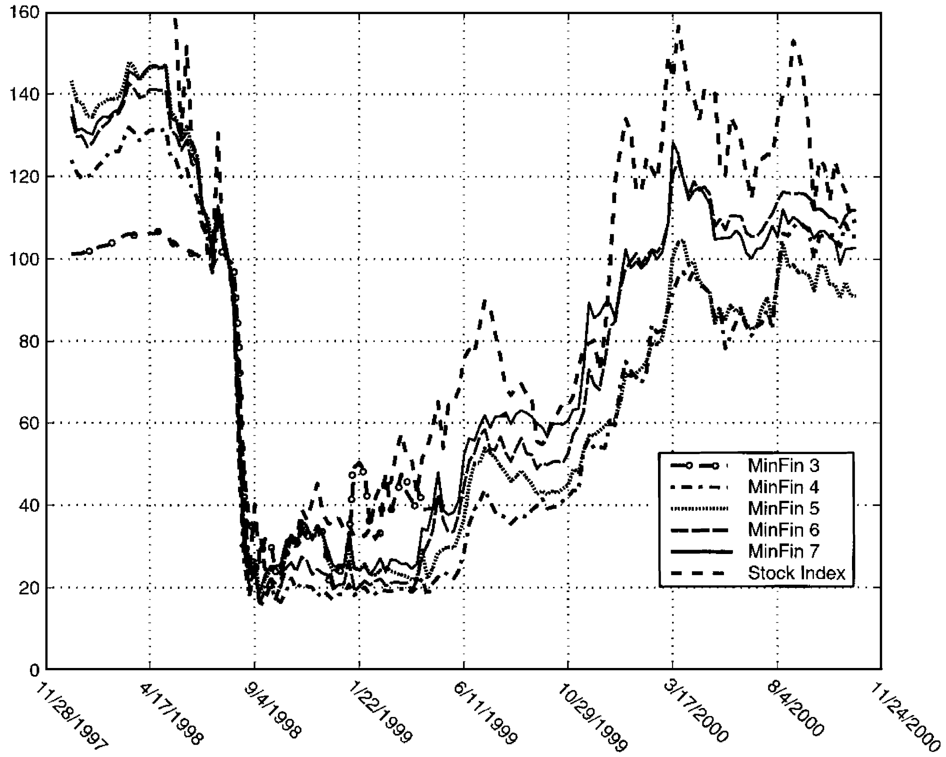


**Figure 1. Russian bond prices.** Prices of Russian MinFin 3 to 7 bonds over the time period from January 2, 1998, to November 17, 2000, normalized so that they all have a market value of 100 at July 31, 1998 (just prior to default). Source: Datastream.