



Inequality in a
monetary
dynamic
macroeco-
nomic
model

M. R. Grasselli

Introduction

Review of
Piketty

Dual Keen
model

Inequality and
speculation

Conclusions

Inequality in a monetary dynamic macroeconomic model

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The book

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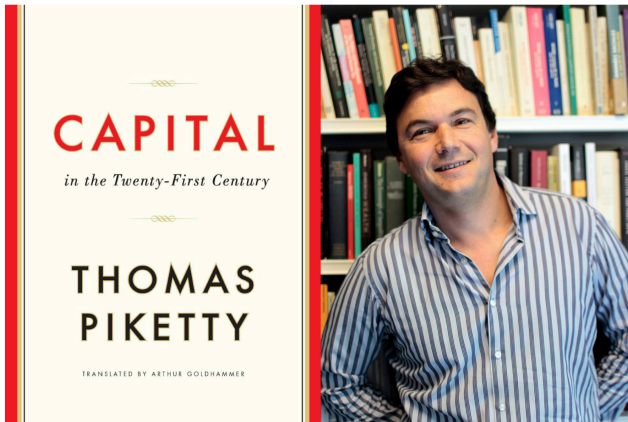
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To put it bluntly, the discipline of economics has yet to get over its childish passion for mathematics and for purely theoretical and often highly ideological speculation, at the expense of historical research and collaboration with the other social sciences.

Piketty's model is not a deterministic system from which he attempts to predict all future economic history, but rather a system of interacting mathematical regularities and patterns, themselves directly measurable from the statistical analysis of historical data, intended to give a good match to empirically observed results, and from which we can then make some predictions about the future by extrapolating the most robust trends and incorporating what we know of present economic conditions. (Dan Kervik, Rugged Egalitarianism)

Key definitions

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- $Y_n = (Y_n - W) + W$ (total income equals capital income plus labor income)
- $r_k = \frac{(Y_n - W)}{pK}$ (rate of return on capital)
- $\alpha_k = \frac{Y_n - W}{Y_n}$ (capital share of total income)
- $\beta_k = \frac{pK}{Y_n}$ (capital-to-income ratio)

Output growth

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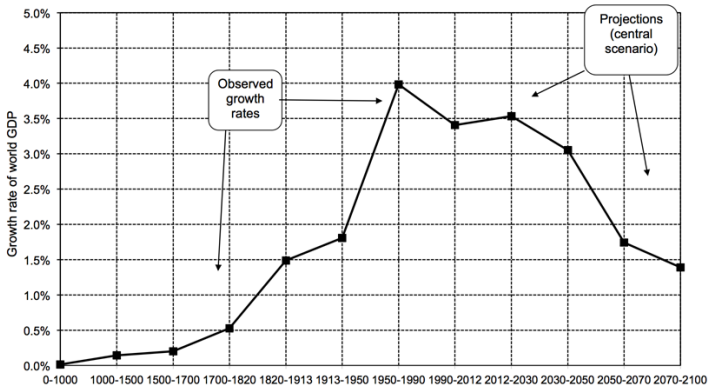
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Figure 2.5. The growth rate of world output from Antiquity until 2100



The growth rate of world output surpassed 4% from 1950 to 1990. If the convergence process goes on it will drop below 2% by 2050. Sources and series: see piketty.pse.ens.fr/capital21c.

Rate of return on capital - Britain

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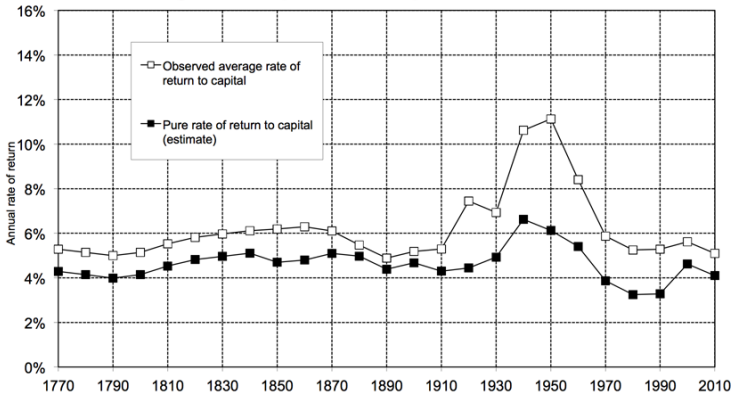
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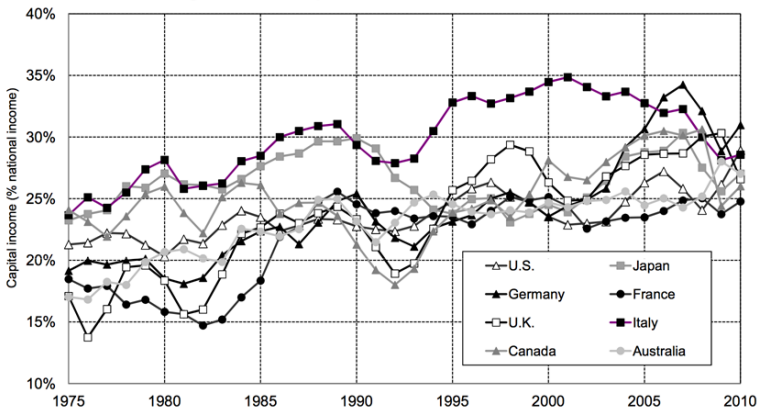
Figure 6.3. The pure return to capital in Britain, 1770-2010



The pure rate of return to capital is roughly stable around 4%-5% in the long run.

Sources and series: see piketty.pse.ens.fr/capital21c.

Figure 6.5. The capital share in rich countries, 1975-2010



Capital income absorbs between 15% and 25% of national income in rich countries in 1970, and between 25% and 30% in 2000-2010. Sources and series: see piketty.pse.ens.fr/capital21c

Capital-to-Income ratio - Britain

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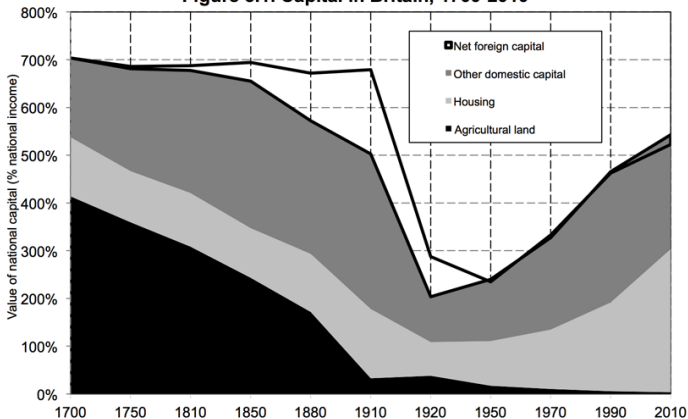
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Figure 3.1. Capital in Britain, 1700-2010



National capital is worth about 7 years of national income in Britain in 1700 (including 4 in agricultural land).

Sources and series: see piketty.pse.ens.fr/capital21c.

The argument in a nutshell

- First Law of Capitalism:

$$\alpha_k = \frac{(Y_n - W)}{Y_n} = \frac{(Y_n - W)}{pK} \frac{pK}{Y_n} = r_k \beta_k$$

- Second Law of Capitalism:

$$\beta_k \rightarrow \frac{s}{g}$$

- Therefore, if $r_k > g$, wealth and income inequality tend to increase in time.

Underpants Gnome's Business Plans

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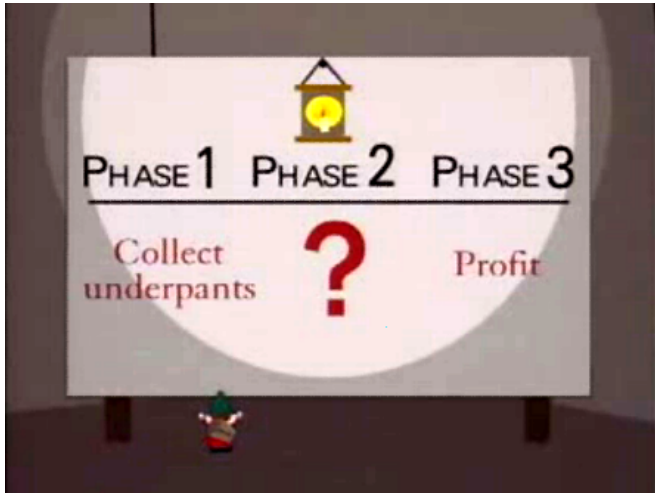
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The inequality $r > g$ implies that wealth accumulated in the past grows more rapidly than output and wages. This inequality expresses a fundamental logical contradiction. The entrepreneur inevitably tends to become a rentier, more and more dominant over those who own nothing but their labor. Once constituted, capital reproduces itself faster than output increases. The past devours the future.

- Validity of the Second Law of Capitalism
- Stability of the relationship $r_k > g$
- Cambridge Capital Controversies
- Representative Agent
- Nevertheless . . .

Capital-to-Income - World



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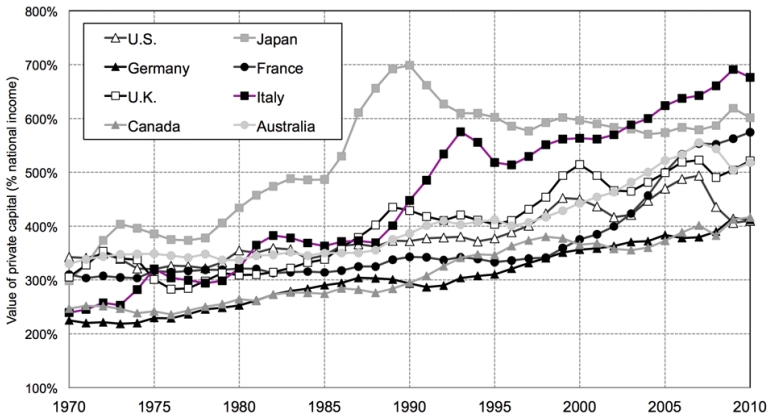
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Figure 5.3. Private capital in rich countries, 1970-2010



Private capital is worth between 2 and 3.5 years of national income in rich countries in 1970, and between 4 and 7 years of national income in 2010. Sources and series: see piketty.pse.ens.fr/capital21c.

Return on capital versus growth

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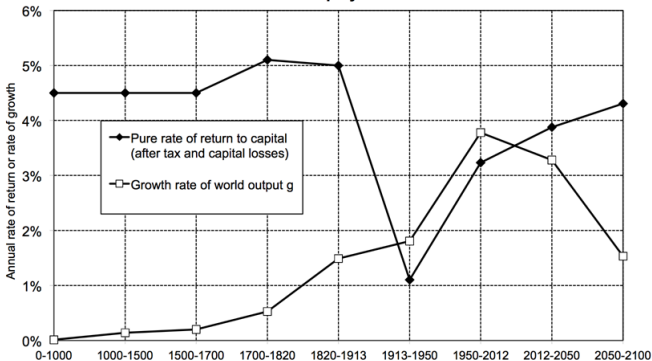
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Figure 10.10. After tax rate of return vs. growth rate at the world level, from Antiquity until 2100



The rate of return to capital (after tax and capital losses) fell below the growth rate during the 20th century, and may again surpass it in the 21st century. Sources and series : see piketty.pse.ens.fr/capital21c

Income inequality - top 1%

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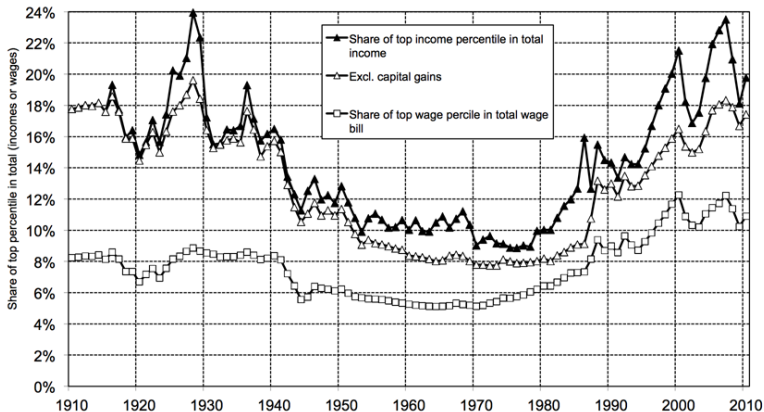
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Figure 8.8. The transformation of the top 1% in the United States



The rise in the top 1% highest incomes since the 1970s is largely due to the rise in the top 1% highest wages. Sources and series: see piketty.pse.ens.fr/capital21c.

Income inequality - top 0.1%

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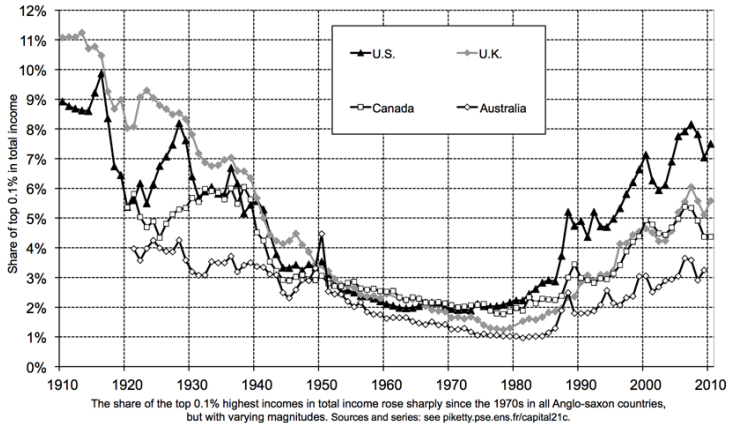
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Figure 9.5. The top 0.1% income share in Anglo-saxon countries, 1910-2010



Wealth inequality

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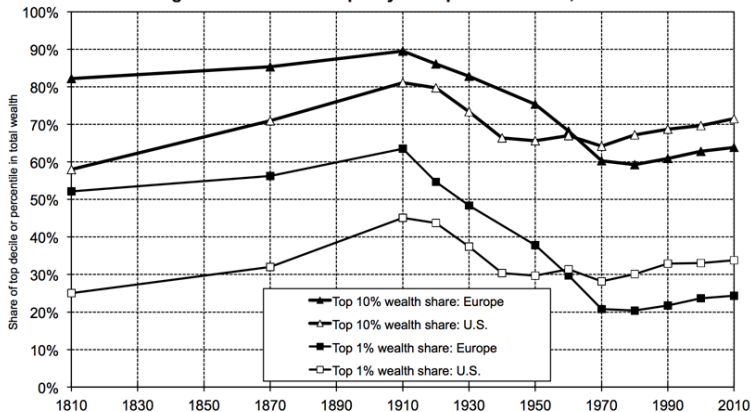
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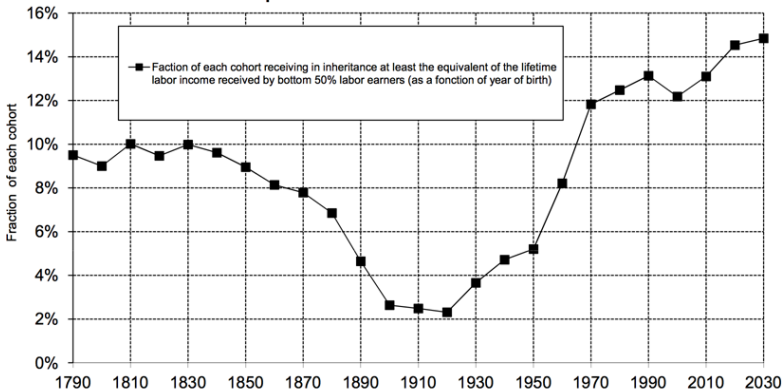
Figure 10.6. Wealth inequality: Europe and the U.S., 1810-2010



Until the mid 20th century, wealth inequality was higher in Europe than in the United States.

Sources and series: see piketty.pse.ens.fr/capital21c.

Figure 11.11. Which fraction of a cohort receives in inheritance the equivalent of a lifetime labor income?



Within the cohorts born around 1970-1980, 12-14% of individuals receive in inheritance the equivalent of the lifetime labor income received by the bottom 50% less well paid workers. Sources and series : see piketty.pse.ens.fr/capita21c

SFC table for the dual Keen model

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Balance Sheet	Households	Firms		Banks	Sum
		current	capital		
Deposits	$+M_h$	$+M_f$		$-M$	0
Loans	$-L_h$			$+L$	0
Capital			$+pK$		pK
Sum (net worth)	X_h	0	X_f	X_b	pK
Transactions					
Consumption	$-pC$	$+pC$			0
Investment		$+pI$	$-pI$		0
Acct memo [GDP]		$[pY]$			
Wages	$+W$	$-W$			0
Interest on deposits	$+rM_h$	$+rM_f$		$-rM$	0
Interest on loans	$-rL_h$			$+rL$	0
Profits		$-\Pi$	$+\Pi_u$		0
Sum	S_h	0	$S_f - pI$	S_b	0
Flow of Funds					
Deposits	$+M_h$	$+M_f$		$-M$	0
Loans	$-L_h$			$+L$	0
Capital			$+pI$		pI
Sum	S_h	0	Π_u	0	pI
Change in Net Worth	S_h	$(S_f + pK - p\delta K)$			$pK + pK$

Table: SFC table for the dual Keen model.

- Let

$$\dot{D}_h = pC - W + rD_h.$$

- Denoting $\omega = W/Y_n$, $d = D_h/Y_n$, assume that consumption is given by $C := c(\omega - rd)Y$ for a function c of disposable income $(\omega - rd)$.
- Letting $I = Y - C$, we have that

$$\dot{K} = Y - C - \delta K = \left(\frac{1 - c(\omega - rd)}{\nu} - \delta \right) K$$

where $\nu = K/Y$ is a constant capital-to-output ratio.

- Assume further a wage-price dynamics of the form

$$\frac{\dot{w}}{w} = \Phi(\lambda) + \gamma \left(\frac{\dot{p}}{p} \right)$$

$$i(\omega) = \frac{\dot{p}}{p} = \eta_p(m\omega - 1),$$

for a constant mark-up factor $m \geq 1$.

- The model can now be described by the following system

$$\begin{cases} \frac{\dot{z}}{z} = \Phi(\lambda) - \alpha - (1 - \gamma)i(\omega) \\ \frac{\dot{\lambda}}{\lambda} = \frac{1-c[\omega-rd]}{\nu} - (\alpha + \beta + \delta) \\ \dot{d} = d \left[r + \delta - \frac{1-c[\omega-rd]}{\nu} - i(\omega) \right] + c[\omega - rd] - \omega. \end{cases}$$

- Analogously to the original Keen model, this model exhibits a good equilibrium characterized by

$$\bar{\omega}_1 = \eta + r \left[\frac{1 - \eta - \nu(\alpha + \beta + \delta)}{\alpha + \beta + i(\bar{\omega}^1)} \right].$$

$$\bar{\lambda}_1 = \Phi^{-1}(\alpha + (1 - \gamma)i(\bar{\omega}^1)).$$

$$\bar{d}_1 = \frac{1 - \eta - \nu(\alpha + \beta + \delta)}{\alpha + \beta + i(\bar{\omega}^1)},$$

where $\eta := c^{-1}(1 - \nu(\alpha + \beta + \delta))$.

- It also exhibits a bad equilibrium of the form $(0, 0, +\infty)$.
- Both equilibria are locally stable for typical parameter values.

Workers versus investors - motivation

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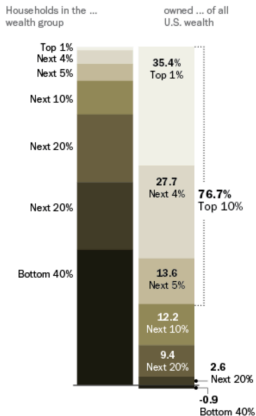
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Distribution of U.S. Wealth, 2010



Source: "The Asset Price Meltdown and the Wealth of the Middle Class," by Edward N. Wolff, NYU (November 2012)

Workers versus investors - modelling

- Consider now two different classes of households, namely workers and investors, with wealth given by

$$X_w = -D_w$$

$$X_i = qS - D_i.$$

- It follows from the budget constraint that

$$\dot{D}_w = pC_w - W + rD_w$$

$$\dot{D}_i = pC_i - r_k pK - rD_w.$$

- Finally, assume that consumption is of the form $C_w = c_w(y_w, x_w)Y$ and $C_i = c_i(y_i, x_i)Y$ for functions c_* of income y_* and wealth x_* satisfying

$$\frac{\partial c_w}{\partial y_w}(\omega - rd_w, x_w) > \frac{\partial c_i}{\partial y_i}(r_k \nu + rd_w, x_i).$$

Return on capital and equilibria

- We assume the firms retain profits according to a constant retention rate s_π , leading to an endogenous return on capital given by

$$r_k = \frac{(1 - s_\pi)\Pi}{pK} = \frac{1 - s_\pi}{\nu}(1 - \omega).$$

- This leads to the modified system

$$\begin{cases} \dot{\omega} = \Phi(\lambda) - \alpha - (1 - \gamma)i(\omega) \\ \dot{\lambda} = \frac{1-c}{\nu} - (\alpha + \beta + \delta) \\ \dot{d}_w = d_w \left[r + \delta - \frac{1-c}{\nu} - i(\omega) \right] + c_w - \omega. \\ \dot{d}_i = d_i \left[\delta - \frac{1-c}{\nu} - i(\omega) \right] + c_i - r_k \nu - rd_w. \end{cases}$$

- As before, the system admits a good equilibrium $(\bar{\omega}, \bar{\lambda}, \bar{d}_w, \bar{d}_i)$ with finite debt levels, and bad equilibria of the form $(0, 0, +\infty, \pm\infty)$.

- The growth rate of real net income $(\omega - rd_w)Y$ for workers is given by

$$g_w = \frac{(\dot{\omega} - rd_w)}{\omega - rd_w} + \frac{\dot{Y}}{Y}.$$

- The growth rate of real net income $(r_k\nu + rd_w)Y$ for investors is

$$g_i = \frac{(1 - s_\pi)\dot{\omega} + rd_w}{r_k\nu + rd_w} + \frac{\dot{Y}}{Y}.$$

- At the good equilibrium, both rates equal $\alpha + \beta$ and the income ratio for the two classes converge to a constant.
- At the bad equilibria, on the other hand, it is clear that both classes of households have zero income asymptotically (since $Y \rightarrow 0$), BUT the ratio of capital income to labour income goes to infinity.

Endogenous portfolio change

- Let θ denote the fraction of the investor's wealth allocated to stocks, that is, $qS = \theta X_i$.
- Assume that

$$\dot{\theta} = \mu(\theta^*(r^e) - \theta) \quad \theta^{*'} > 0, \mu > 0$$

where $\theta^*(\cdot)$ is the desired share of equity and r^e is the expected rate of return on equity.

- Furthermore, assume that expectations are adaptive, namely,

$$\dot{r}^e = \rho(r_k - r^e) \quad \rho > 0.$$

- Similarly to the introduction of Ponzi speculation in the Keen model, this reduces the basin of attraction for the good equilibrium.

- We provided a stock-flow consistent model for debt dynamics of workers and investors.
- When the economy converges to an equilibrium with finite debt ratios, the income ratio between the two classes is constant.
- Increasing income (and wealth) inequality is a signature of convergence to the bad equilibrium with infinite debt ratios.
- In future work we explore the effects of default and of migration between classes a la Acemoglu (2014).
- OBRIGADO!