Investment, abandonment, mothballing and reactivation in incomplete markets

A Real Options Approach

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May 26, 2006

Strategic Decision Making

Suppose we want to assign *monetary values* to the strategic decision to:

- create a new firm;
- invest in a new project;
- start a real estate development;
- finance R&D;
- abandon a non-profitable project;
- temporarily suspend operations under adverse conditions and reactive them when conditions improve.

Valuation Elements

In all of the previous problems, we can identify the following common elements:

- intrinsic value;
- uncertainty about the future;
- some degree of irreversibility;
- timing flexibility;
- managerial flexibility.

Net Present Value

- Net Present Values takes into account the *intrinsic* advantages of a given investment when compared to capital markets.
- This are essentially due to market imperfections, such as entry barriers, product differentiation, economy of scale, etc...
- For instance, denoting the expected present value of *future cash flows* of a given project by $\widetilde{V}$ and the corresponding *sunk cost* by I, then its NPV is

$$\text{NPV} = \widetilde{V} - I$$

- Therefore, the decision rule according to this NPV is to invest whenever $\widetilde{V} > I$.

The Real Options Approach

- If we view the project value V as an underlying asset, then an investment opportunity with a sunk cost I is the *formal* analogue of an American call option on $\widetilde{V}$ with strike price I.
- The Real Options Approach then applies techniques used for *financial* options to determined the value $\widetilde{C}$ for the option to invest.
- Therefore, an investors possessing an opportunity with value C will invest only when

$$\widetilde{V} - I > \widetilde{C}.$$

- This then results in *higher* exercise thresholds, taking into account the *value of waiting*.

Successes and Limitations

- According to a recent survey, 26% of CFOs in North America "always or almost always" consider the value of real options in projects.
- This is due to familiarity with the option valuation paradigm in financial markets and its lessons.
- *But* most of the literature in Real Options is based on one or both of the following assumptions: (1) *infinite time horizon* and (2) perfectly correlated *spanning asset* .
- Though some problems have long time horizons (30 years or more), most strategic decisions involve much shorter times.
- The vast majority of underlying projects are *not* perfectly correlated to any asset traded in financial markets.

Alternatives

- The use of well–known numerical methods (e.g binomial trees of finite differences) allowed many authors to successfully drop the infinite time horizon hypothesis.
- As for the spanning asset assumption, the absence of perfect correlation with a financial asset leads to an *incomplete market*.
- Replication arguments can no longer be applied to value managerial opportunities.
- Instead, one needs to rely on *risk preferences*.
- The most widespread way to do this in the strategic decision making literature is to introduce an *internal rate of return*, which replaces the risk–free rate, and use dynamic programming.
- This approach lacks the intuitive understanding of opportunities as *options*.

Utility-based methods

- We treat an investment opportunity as an option on a *non–traded asset* and price it using the framework of *indifference pricing*.
- For investments with a fixed exercise date (European option), this problem was treated, for instance, in Hobson and Henderson (2002).
- For early exercise investment (American option), the problem was solved in Herderson (2005) for the case of *infinite* time horizon..
- An different utility–based framework (not using indifference pricing), was treated in Hugonnier and Morellec (2004), using the effect of shareholders control on the wealth of a risk averse manager.

A one–period investment model

- Consider the two–factor market where the *discounted* project value V and the *discounted* a correlated traded asset S following:

$$(S_T, V_T) = \begin{cases} (uS_0, hV_0) & \text{with probability } p_1, \\ (uS_0, \ell V_0) & \text{with probability } p_2, \\ (dS_0, hV_0) & \text{with probability } p_3, \\ (dS_0, \ell V_0) & \text{with probability } p_4, \end{cases} \tag{1}$$

  where $0 < d < 1 < u$ and $0 < \ell < 1 < h$, for positive initial values S_0, V_0 and historical probabilities p_1, p_2, p_3, p_4.
- Let the risk preferences be specified through an exponential utility $U(x) = -e^{-\gamma x}$.
- An investment opportunity is model as an option with *discounted* payoff $C_t = (V - e^{-rt} I)^+$, for $t = 0, T$.

European Indifference Price

- Without the opportunity to invest in the project V, a rational agent with initial wealth x will try to solve the optimization problem
$$u^0(x) = \max_H E[U(X_T^x)], \tag{2}$$
where
$$X_T^x = \xi + HS_T = x + H(S_T - S_0). \tag{3}$$
is the wealth obtained by keeping ξ dollars in a risk–free cash account and holding H units of the traded asset S.
- An agent with initial wealth x who pays a price π for the opportunity to invest in the project will try to solve the modified optimization problem
$$u^C(x - \pi) = \max_H E[U(X_T^{x-\pi} + C_T)] \tag{4}$$
- The *indifference price* for the option to invest in the final period as the amount π^C that solves the equation
$$u^0(x) = u^C(x - \pi). \tag{5}$$

Explicit solution

Denoting the two possible pay-offs at the terminal time by C_h and C_ℓ, the European indifference price defined in (5) is given by

$$\pi^C = g(C_h, C_\ell) \tag{6}$$

where, for fixed parameters $(u, d, p_1, p_2, p_3, p_4)$ the function $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is given by

$$\begin{aligned} g(x_1, x_2) \quad &= \quad \frac{q}{\gamma} \log \left(\frac{p_1 + p_2}{p_1 e^{-\gamma x_1} + p_2 e^{-\gamma x_2}} \right) \\ &\quad + \frac{1-q}{\gamma} \log \left(\frac{p_3 + p_4}{p_3 e^{-\gamma x_1} + p_4 e^{-\gamma x_2}} \right), \end{aligned} \tag{7}$$

with

$$q = \frac{1-d}{u-d}.$$

Early exercise

- When investment at time $t = 0$ is allowed, it is clear that immediate exercise of this option will occur whenever its *exercise value* $(V_0 - I)^+$ is larger than its *continuation value* given by π^C.
- That is, from the point of view of this agent, the value at time zero for the opportunity to invest in the project either at $t = 0$ or $t = T$ is given by

$$C_0 = \max\{(V_0 - I)^+, g((hV_0 - e^{-rT} I)^+, (\ell V_0 - e^{-rT} I)^+)\}.$$

A multi–period model

- Consider now a continuous-time two–factor market of the form

$$
\begin{aligned}
dS_t &= (\mu_1 - r)S_t dt + \sigma_1 S_t dW \\
dV_t &= (\mu_2 - r)V_t dt + \sigma_2 V_t(\rho dW + \sqrt{1-\rho^2})dZ.
\end{aligned}
$$

- We want to approximate this market by a discrete–time processes (S_n, V_n) following the one–period dynamics (1).
- This leads to the following choice of parameters:

$$
\begin{aligned}
u &= e^{\sigma_1\sqrt{\Delta t}}, \qquad h = e^{\sigma_2\sqrt{\Delta t}}, \\
d &= e^{-\sigma_1\sqrt{\Delta t}}, \qquad \ell = e^{-\sigma_2\sqrt{\Delta t}}, \\
p_1 + p_2 &= \frac{e^{(\mu_1 - r)\Delta t} - d}{u - d}, \quad p_1 + p_3 = \frac{e^{(\mu_2 - r)\Delta t} - \ell}{h - \ell} \\
\rho\sigma_1\sigma_2\Delta t &= (u-d)(h-\ell)[p_1 p_4 - p_2 p_3],
\end{aligned}
$$

supplemented by the condition $p_1 + p_2 + p_3 + p_4 = 1$.

Grid Values

- Instead a triangular tree for project values, we consider a $(2M+1) \times N$ rectangular grid whose repeated columns are given by

$$V^{(i)} = h^{M+1-i} V_0, \qquad i = 1, \ldots, 2M+1. \tag{8}$$

  This range from $(h^M V_0)$ to $(\ell^M V_0)$, respectively the highest and lowest achievable discounted project values starting from the middle point V_0 with the multiplicative parameter $h = \ell^{-1} > 1$.
- The parameter M should be chosen so that such highest and lowest values are comfortably beyond the range of project values that can be reached during the time interval $[0, T]$ with reasonable probabilities (say four standard deviations)
- Then each realization for the discrete-time process V_n following the dynamics (1) can then be thought of as a path over this grid.

Option pricing on the grid

- We determine the *discounted* value of the option to invest on the project can is a function C_{in} on this grid.
- We start by with the boundary conditions:

$$
\begin{array}{rcll}
C_{iN} & = & (V^{(i)} - e^{-rT} I)^+, & i = 1, \dots, 2M+1, \\
C_{1n} & = & V^{(1)} - e^{-rn\Delta_t}, & n = 0, \dots, N, \\
C_{2M+1,n} & = & 0, & n = 0, \dots, N.
\end{array}
$$

- Values in the interior of the grid are then obtained by backward induction as follows:

$$
C_{in} = \max\left\{ (V^{(i)} - e^{-rn\Delta t} I)^+, g\left(C_{i+1,n+1}, C_{i-1,n+1}\right) \right\}. \quad (9)
$$

- For each time t_n, the *exercise threshold* V_n^* is defined as the project value for which the exercise value becomes higher than its continuation value.

Numerical Experiments

- We now investigate how the exercise threshold varies with the different model parameters.
- The fixed parameters are

$$
\begin{aligned}
I &= 1, \quad r = 0.04, \quad T = 10 \\
\mu_1 &= 0.115, \quad \sigma_1 = 0.25, \quad S_0 = 1 \\
\sigma_2 &= 0.2, \quad V_0 = 1
\end{aligned}
$$

- Given these parameters, the CAPM equilibrium expected rate of return on the project for a given correlation ρ is

$$
\bar{\mu}_2 = r + \rho \left(\frac{\mu_1 - r}{\sigma_1} \right) \sigma_2. \tag{10}
$$

- The difference $\delta = \bar{\mu}_2 - \mu_2$ is the *below–equilibrium rate–of–return shortfall* and plays the role of a dividend rate paid by the project, which we fix at $\delta = 0.04$.

Known Thresholds

- In the limit $\rho \to \pm 1$ (complete market), the closed–form expression for the investment threshold obtained in the case $T = \infty$ gives $V^*_{DP} = 2$.
- This should be contrasted with the NPV criterion (that is, invest whenever the net present value for the project is positive) which in this case gives $V^*_{NPV} = 1$.
- The limit $\gamma \to 0$ in our model corresponds to the McDonald and Siegel (1986) threshold, obtained by assuming that investors are averse to market risk but neutral towards idiosyncratic risk.
- For our parameters, the adjustment to market risks is accounted by CAPM and this threshold coincides with market risk is threshold is $V^*_{DP} = 2$

Dependence with Correlation and Risk Aversion

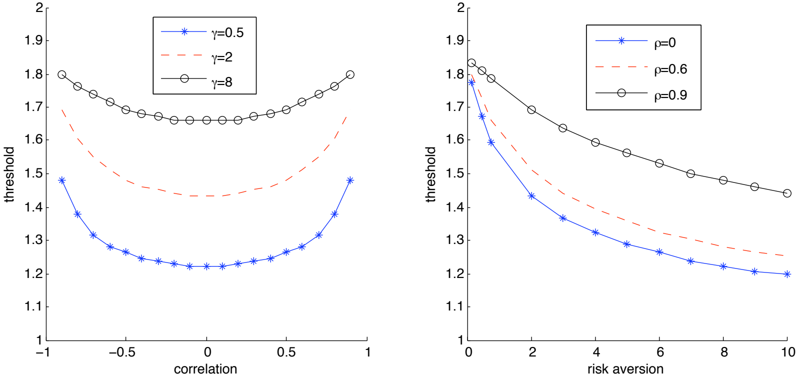

Figure: Exercise threshold as a function of correlation and risk aversion.

Dependence with Correlation and Risk Aversion

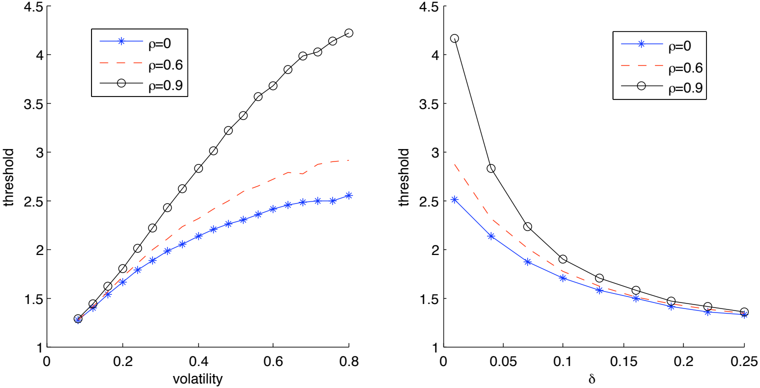

Figure: Exercise threshold as a function of volatility and dividend rate.

Dependence with Time to Maturity

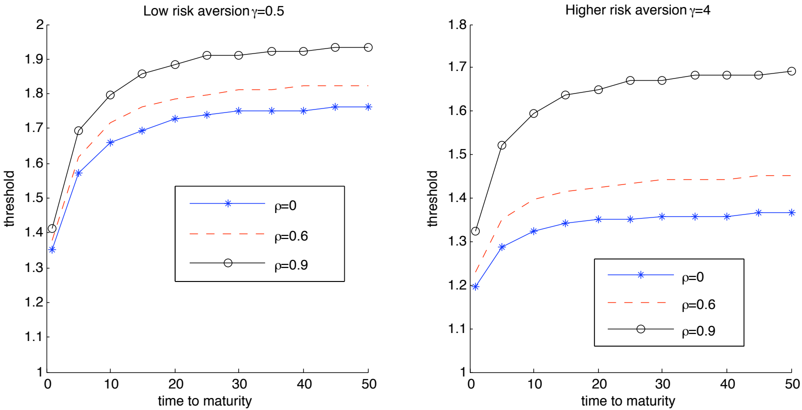

Figure: Exercise threshold as a function of time to maturity.

Values for the option to invest

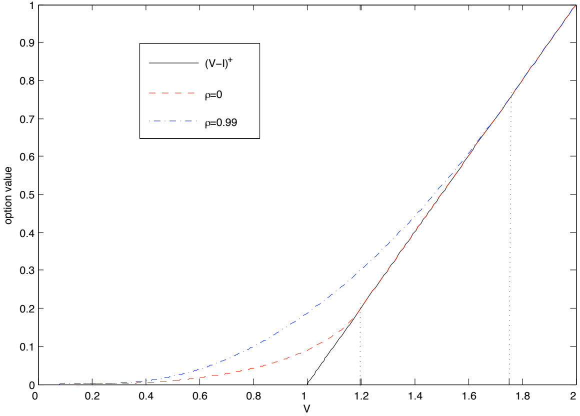

Figure: Option value as a function of underlying project value. The threshold for $\rho = 0$ is 1.1972 and the one for $\rho = 0.99$ is 1.7507.

The abandonment option

- The previous framework ignores the possibility of negative cash flows arising from the active project, for instance, when operating costs exceed the revenue.
- Instead of the project value, we need to model two other underlying variables: the (random) output cash flow rate P_t, governed by

$$dP_t = (\mu_2 - r)P_t dt + \sigma_2 P_t(\rho dW + \sqrt{1-\rho^2})dZ$$

  and the (fixed) operating cost rate C.
- The cash expected value at time t of future cash flows for the project is then

$$P_t/\delta - C/r.$$

- We then have to consider the option to abandon the project when such cash flows become too negative.

Suspension, Reactivation and Scrapping

- Instead of completely abandoning the project, we might have the option to “mothball” it by paying a sunk cost E_M and a maintenance rate $m < C$.
- Once prices for the output become favorable again, we have the option to reactive the project by paying a sunk cost $R < I$.
- Finally, if prices drop too much, we have the option to completely abandon the project by paying a sunk cost S (which could be negative, corresponding to a “scrap value”).
- As before, the decisions to invest, mothball, reactivate and scrap are triggered by the price thresholds $P_S < P_M < P_R < P_H$.

Project values and options

- Let us denote the value of an idle project by F^0, an active project by F^1 and a mothballed project by F^M. as the value of a mothballed project.
- Then

$$\begin{aligned} F^0 &= \text{option to invest at cost } I \\ F^1 &= \text{cash flow } + \text{option to mothball at cost } E_M \\ F^M &= \text{cash flow } + \text{option to reactivate at cost } R_A \\ &\quad + \text{option to scrap at cost } S \end{aligned}$$

- We obtain its value on the grid using the recursion formula

$$F^k(i,j) = \max\{\text{continuation value, possible exercise values}\}.$$

Boundary values

- At the bottom of the grid we have, for all $j = 0, \dots, N$:

$$\begin{aligned} F^0(2M+1, j) &= 0 \\ F^1(2M+1, j) &= -(S + E_M) \\ F^M(2M+1, j) &= -S \end{aligned}$$

- Similarly, at the top of the grid we should have, for all $j = 0 \dots, N$:

$$\begin{aligned} F^0(1, j) &= P(1)/\delta C/r - I \\ F^1(1, j) &= P(1)/\delta - C/r \\ F^M(1, j) &= P(1)/\delta - C/r - R \end{aligned}$$

- Finally, at the final time we have, for $i = 1, \dots 2M+1$:

$$\begin{aligned} F^0(i, N) &= \max(0, P(i)/\delta - C/r - I) \\ F^1(i, N) &= \max(P(i)/\delta - C/r, -m/r - E_M, -S - E_M) \\ F^M(i, N) &= \max(-m/r, P(i)/\delta - C/r - R, -S) \end{aligned}$$

Idle and Mothballed Phases

- For the idle phase we have

$$\begin{aligned}
\text{cont}^0(i,j) &= g(F^0(i-1,j+1), F^0(i+1,j+1) \\
\text{exer}^1(i,j) &= g(F^1(i-1,j+1) - P(i-1)/\delta + C/r, \\
& \quad F^1(i+1,j+1) - P(i+1)/\delta + C/r) \\
&+ P(i)/\delta - C/r) - I
\end{aligned}$$

- We then move to the mothballed phase, for which

$$\begin{aligned}
\text{cont}^M(i,j) &= -m/r + g(F^M(i-1,j+1) + m/r, \\
& \quad F^M(i+1,j+1) + m/r) \\
\text{exer}^1(i,j) &= P(i)/\delta - C/r - R + \\
& \quad g(F^1(i-1,j+1) - P(i-1)/\delta + C/r), \\
& \quad F^1(i+1,j+1) - P(i+1)/\delta + C/r)] \\
\text{exer}^0(i,j) &= -S + g(F^0(i-1,j+1), F^0(i+1,j+1))
\end{aligned}$$

Active phase

- For the active phase, we have the following:

$$
\begin{aligned}
\mathrm{cont}^1(i,j) &= P(i)/\delta - C/r) + \\
&\quad g(F^1(i-1,j+1) - P(i-1)/\delta + C/r, \\
&\quad F^1(i+1,j+1) - P(i+1)/\delta + C/r)] \\
\mathrm{exer}^0(i,j) &= g(F^0(i-1,j+1), F^0(i+1,j+1)) - S - E_M \\
\mathrm{exer}^M(i,j) &= -m/r - E_M + g(F^M(i-1,j+1) + m/r, \\
&\quad F^M(i+1,j+1) + m/r)
\end{aligned}
$$