A Stochastic Extension of the Keen-Minsky Model for Financial Fragility

M. R. Grasselli

Sharcnet Chair in Financial Mathematics
Mathematics and Statistics - McMaster University
Joint work with B. Costa Lima

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Outline

Dynamic General Equilibrium views

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- Money is neutral in its effect on real variables.
- Largely ignores uncertainty by simply subtracting risk premia from all risky returns and treat them as risk-free.
- All variables are assumed to be simultaneously in equilibrium.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information.

The law of demand

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- The 'Hicksian compensated demand' argument then shows that individual demand curves slope downwards.
- Does this generalize to aggregate demand ?

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- This implicitly assumes that the income of each individual is fixed *and* that their purchases do not alter market price.
- But in a closed economy, neither condition is satisfied !

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- For Crusoe himself, his income rises as the price of bananas rise, but so does the cost of producing more bananas: there is no way to know which effect is higher.
- As shown by Gorman (1953), the aggregate demand function *can take any shape at all* !!!

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- He goes on to say that "The necessary and sufficient condition quoted above is intuitively reasonable. It says, in effect, that an extra unit of purchasing power should be spent in the same way no matter to whom it is given."
- In a different direction, Samuelson (1956) proves that the law of demand "apply to all of society *if optimal reallocation of income can be assumed to keep the ethical worth of each person's marginal dollar equal*".

The SMD theorems

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- Second, with general preferences, even if the distribution of income is fixed, market demand functions need not satisfy in any way the classical restrictions which characterize consumer demand functions.
- The utility hypothesis tell us nothing about market demand unless it is augmented by additional requirements.

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- Financial institutions determine the way funds are available for ownership of capital and production.
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- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.

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- Euphoria becomes a panic.
- "Stability - or tranquility - in a world with a cyclical past and capitalist financial institutions is destabilizing".

Goodwin Model (1967) - Assumptions

- Assume that

$$\begin{aligned} N(t) &= N_0 e^{\beta t} && \text{(total labour force)} \\ a(t) &= a_0 e^{\alpha t} && \text{(productivity per worker)} \\ Y(t) &= \nu K(t) = a(t)L(t) && \text{(total yearly output)} \end{aligned}$$

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- Assume further that

$$
\begin{aligned}
\dot{\mathrm{w}} &= \Phi(\lambda)\mathrm{w} && \text{(Phillips curve)} \\
\dot{K} &= (Y - \mathrm{w}L) - \delta K && \text{(Say's Law)}
\end{aligned}
$$

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}L}{Y} = \frac{\mathrm{w}}{a} \quad \text{(wage share)}$$
$$\lambda = \frac{L}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

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- It then follows that

$$\dot{\omega} = \omega(\Phi(\lambda) - \alpha)$$
$$\dot{\lambda} = \lambda\left(\frac{1-\omega}{\nu} - \alpha - \beta - \delta\right)$$

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- Then $(\overline{\omega}_0, \overline{\lambda}_0) = (0,0)$ is a saddle point and the only other equilibrium

$$(\overline{\omega}_1, \overline{\lambda}_1) = \left(1 - \nu(\alpha+\beta+\delta), \Phi^{-1}(\alpha)\right)$$

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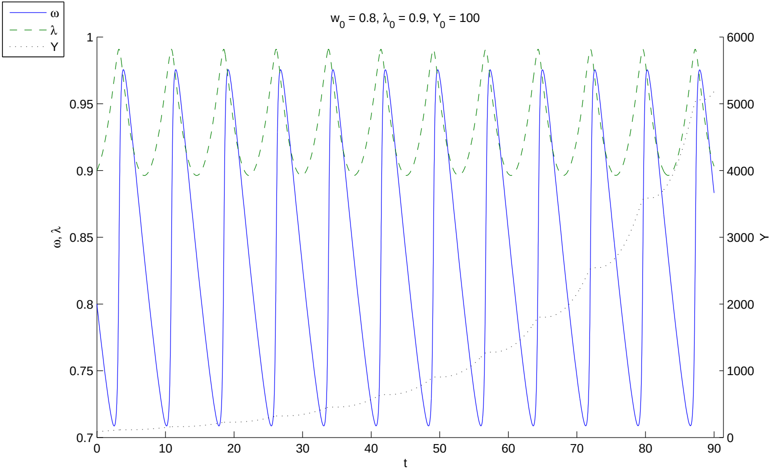
- Moreover

$$g(\overline{\omega}_1) := \frac{\dot{Y}}{Y}(\overline{\omega}_1) = \frac{1-\overline{\omega}_1}{\nu} - \delta = \alpha + \beta,$$

Example 1: Goodwin model

Example 1 (continued): Goodwin model



Introducing a financial sector (Keen 1995)

- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

where $\kappa(\cdot)$ is $C^1(-\infty, \infty)$ satisfying

$$\kappa'(\pi) > 0 \text{ on } (-\infty, \infty)$$

$$\lim_{\pi \to -\infty} \kappa(\pi) = \kappa_0 < \nu(\alpha + \beta + \delta) < \lim_{\pi \to +\infty} \kappa(\pi)$$

$$\lim_{\pi \to -\infty} \pi^2 \kappa'(\pi) = 0.$$

Accordingly, total output evolves as

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- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y$$

Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$\begin{aligned} \dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\ \dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\ \dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega) \end{aligned} \tag{1}$$

Good equilibrium

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- We verify that

$$\overline{\omega}_1 = 1 - \overline{\pi}_1 - r\frac{\nu(\alpha+\beta+\delta) - \overline{\pi}_1}{\alpha+\beta}$$

$$\overline{\lambda}_1 = \Phi^{-1}(\alpha)$$

$$\overline{d}_1 = \frac{\nu(\alpha+\beta+\delta) - \overline{\pi}_1}{\alpha+\beta}$$

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- Moreover

$$g(\overline{\omega}_1, \overline{d}_1) = \frac{\kappa(1-\overline{\omega}_1 - r\overline{d}_1)}{\nu} - \delta = \alpha+\beta.$$

Keen model - Explosive debt

- If we rewrite the system with the change of variables $u = 1/d$, we obtain

$$
\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-r/u)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{u} &= u\left[\frac{\kappa(1-\omega-r/u)}{\nu} - r - \delta\right] - u^2\left[\kappa(1-\omega-r/u) - (1-\omega)\right].
\end{aligned}
\tag{2}
$$

Keen model - Explosive debt

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\dot{u} &= u\left[\frac{\kappa(1-\omega-r/u)}{\nu} - r - \delta\right] - u^2\left[\kappa(1-\omega-r/u) - (1-\omega)\right].
\end{aligned}
$$

- We now see that $(0,0,0)$ is an equilibrium of (2) corresponding to the point

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2) = (0, 0, +\infty)$$

for the original system.

Keen model - Local stability

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- The good equilibrium $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1)$ is stable if and only if

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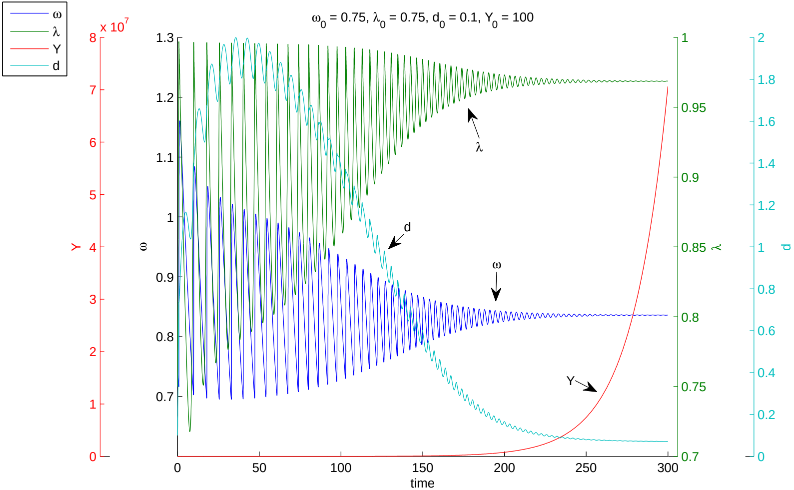
- The point $(0,0,0)$ is a stable equilibrium for (2) if and only if

$$\frac{\kappa_0}{\nu} - \delta < r.$$

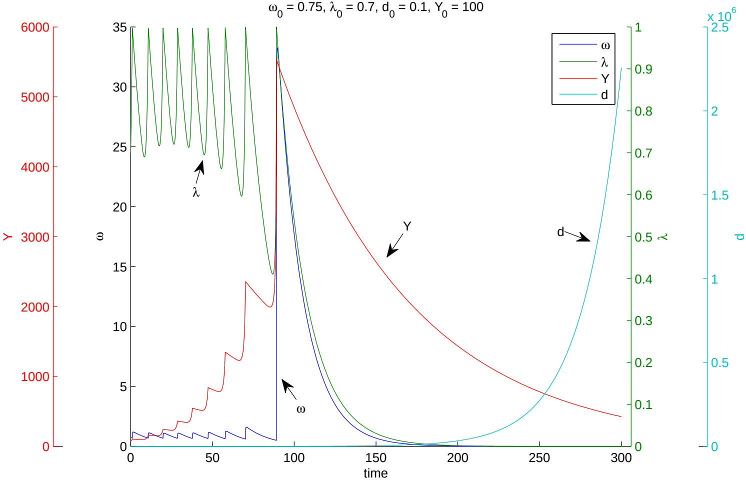
Example 2 : convergence to the good equilibrium in a Keen model

Example 2 (continued): convergence to the good equilibrium in a Keen model



Example 3: explosive debt in a Keen model

Example 3 (continued): explosive debt in a Keen model



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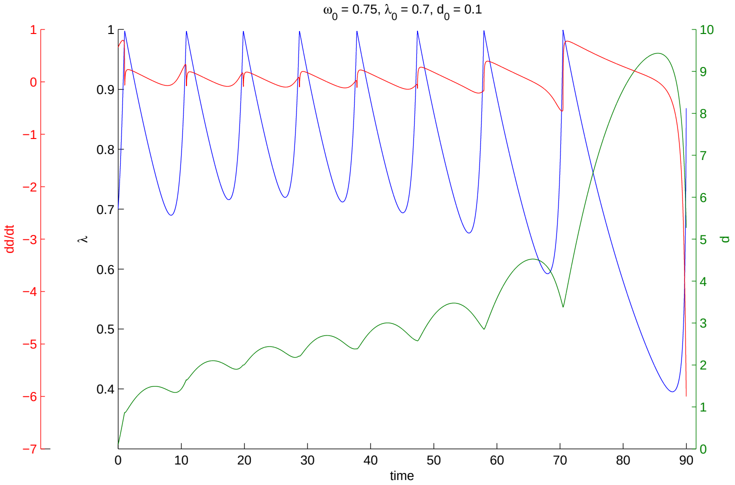
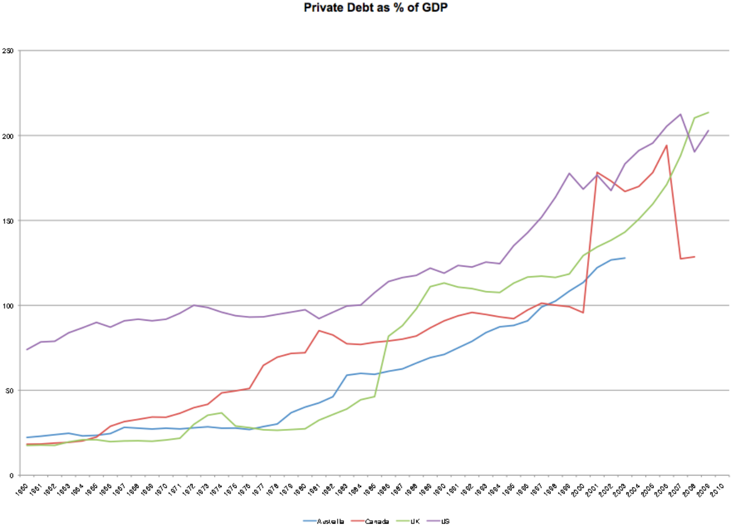
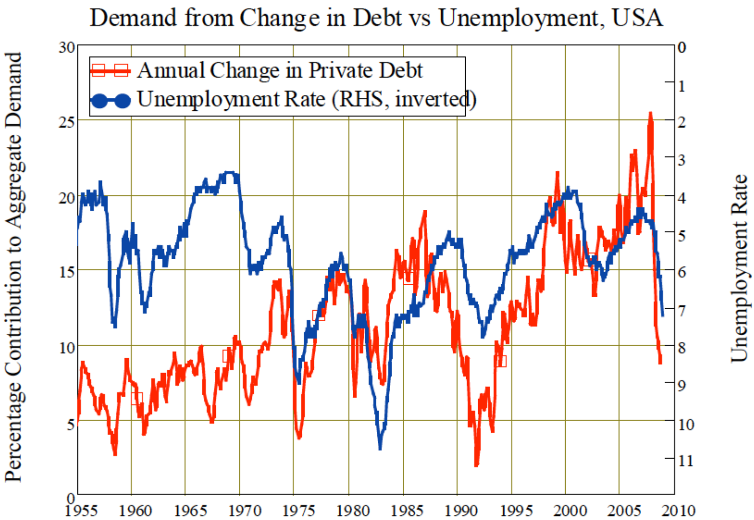


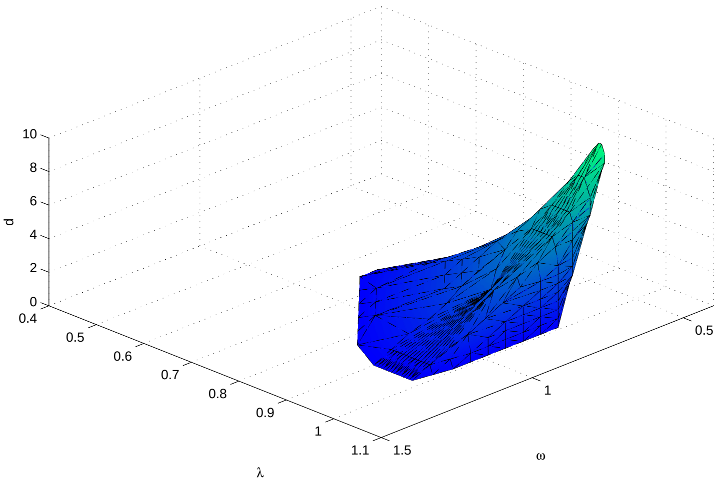
Data detour: debt



Data detour: debt and employment



Basin of convergence for Keen model



Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y + P$$
$$\dot{P} = \Psi(g(\omega,d)P$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g = \frac{\kappa(1-\omega-rd)}{\nu} - \delta.$$

Ponzi financing - Differential equations

With Ponzi financing the dynamical system becomes

$$\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega) + p \\
\dot{p} &= p\left[\Psi\left(\frac{\kappa(1-\omega-rd)}{\nu} - \delta\right) - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right]
\end{aligned} \tag{3}$$

Ponzi financing - Equilibria and stability

- We find that $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1, 0)$ is a stable equilibrium iff

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- Introducing $u = 1/d$ we find that

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2, \overline{p}) = (0, 0, +\infty, 0)$$

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$$\Psi(g_0) < g_0.$$

Ponzi financing - Equilibria and stability

- We find that $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_1, 0)$ is a stable equilibrium iff

$$\Psi(\alpha + \beta) < \alpha + \beta.$$

- Introducing $u = 1/d$ we find that

$$(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_2, \overline{p}) = (0, 0, +\infty, 0)$$

is stable iff

$$\Psi(g_0) < g_0.$$

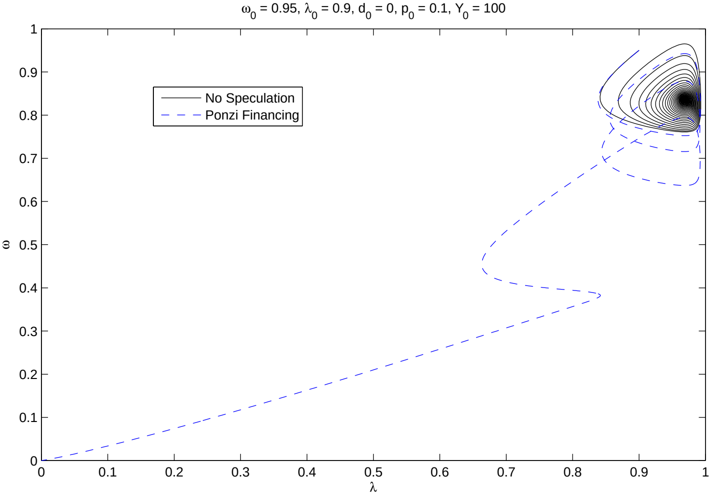
- Moreover, introducing , $x = 1/p$ and $v = p/d$ we find that

$$(\overline{\omega}_3, \overline{\lambda}_3, \overline{d}_3, \overline{p}) = (0, 0, +\infty, +\infty)$$

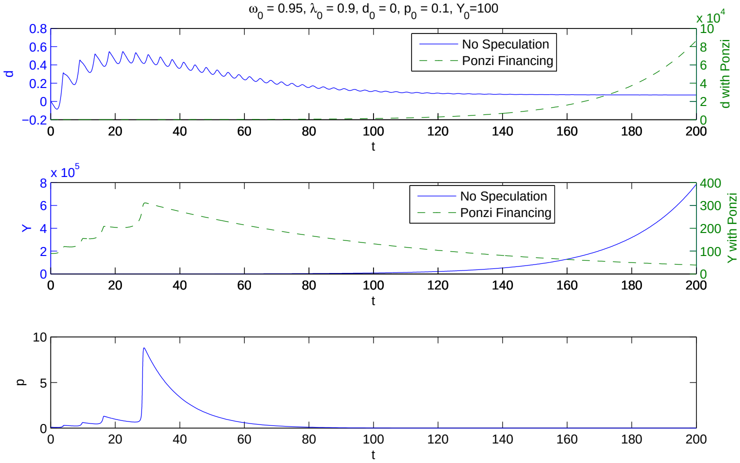
is stable iff

$$g_0 < \Psi(g_0) < r.$$

Example 4: effect of Ponzi financing



Example 4 (continued): effect of Ponzi financing



Introducing a government sector

- A final extension proposed by Keen (echoing Minsky) consists of adding government spending and taxation into the original system according to

$$\dot{G} = \Gamma(\lambda)Y$$
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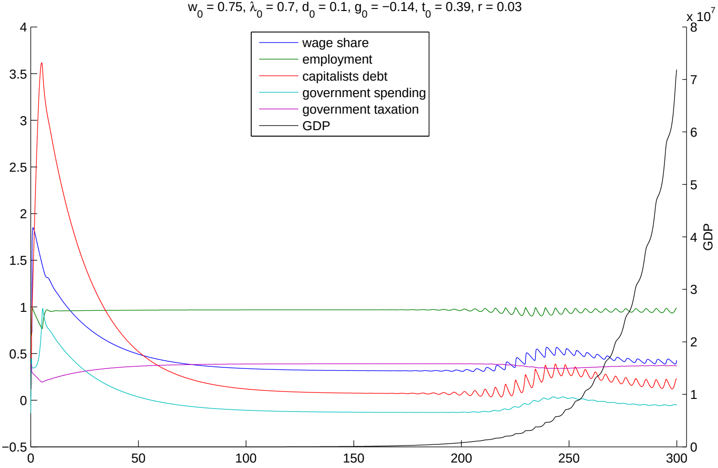
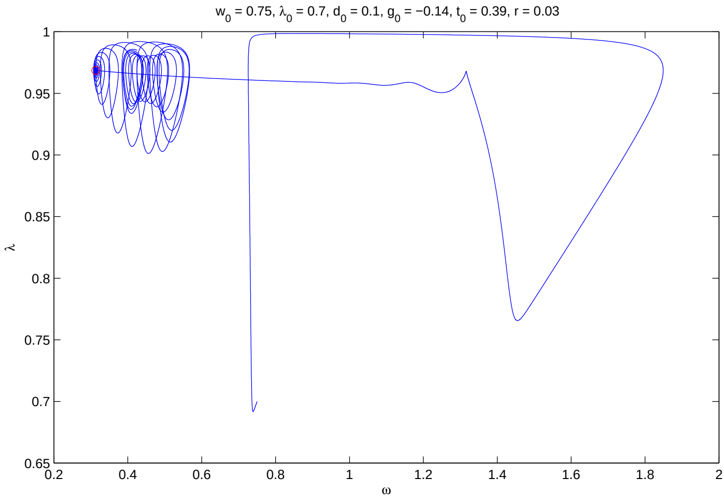
- The new 5-dimensional system displays more local fluctuations, but no breakdown for the same initial conditions as before.

Example 5: stabilizing government



Example 5 (continued): stabilizing government



Stock prices

- Consider a stock price process of the form

$$\frac{dS_t}{S_t} = r_b dt + \sigma dW_t + \gamma \mu_t dt - \gamma dN^{(\mu_t)}$$

where N_t is a Cox process with stochastic intensity $\mu_t = M(p(t))$.

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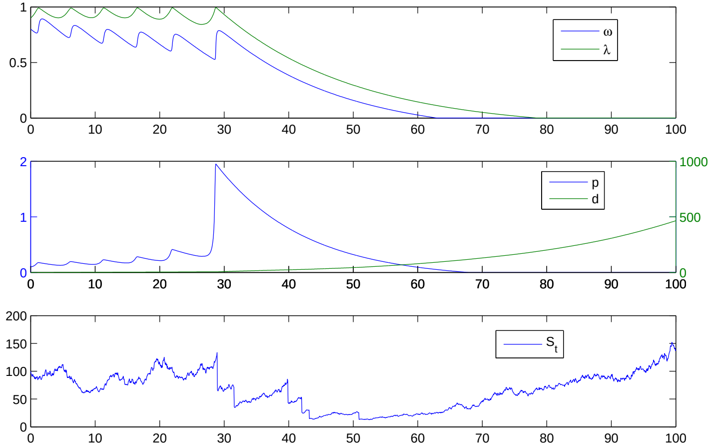
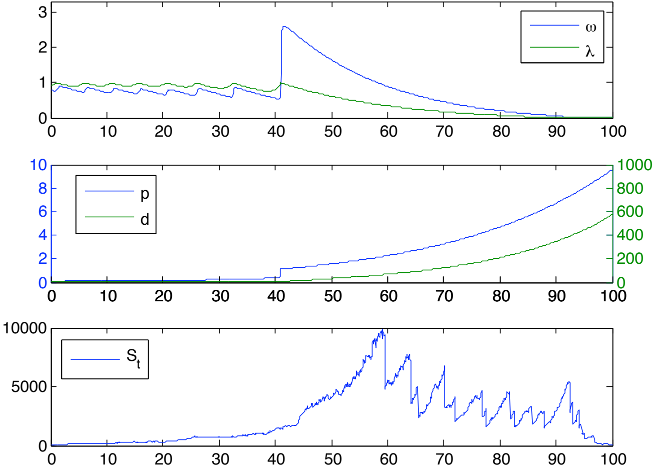
- The interest rate for private debt is modelled as $r_t = r_b + r_p(t)$ where

$$r_p(t) = \rho_1 (S_t + \rho_2)^{\rho_3}$$

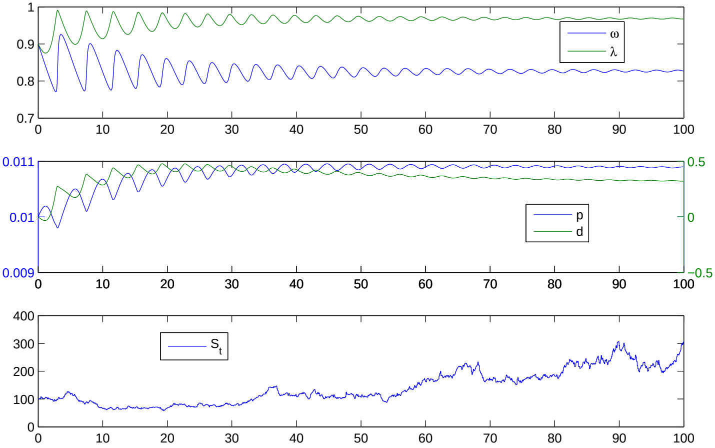
Example 6: stock prices, explosive debt, zero speculation



Example 6: stock prices, explosive debt, explosive speculation



Example 6: stock prices, finite debt, finite speculation



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- Model prices for capital goods P_k and commodities P_c explicitly (Kaleckian mark-up theory, inflation, etc)
- Calibrate to macroeconomic time series.

Concluding thoughts

- Solow (1990): The true test of a simple model is whether it helps us to make sense of the world. Marx was, of course, dead wrong about this. We have changed the world in all sorts of ways, with mixed results; the point is to interpret it.

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- Schumpeter (1939): Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them.