Math 1A03 Fall 2011–12 Practice version of Midterm 1

1) No partial credit will be given on this question.

a) Find the derivative of $f(x) = \arctan(1 + x^3)$. Do not simplify your answer.

b) Find the derivative of $f(x) = \dfrac{2^x + 10}{\cos x}$. Do not simplify your answer.

c) Find the derivative of $f(x) = \ln(2x) \sec x$. Do not simplify your answer.

d) If $g(x) = (f(x) + x)^2$, where $f(3) = 6, f'(3) = -8, f'(6) = 10$, find $f'(3)$.

e) Find the derivative of the function $f(x) = \displaystyle\int_0^{x^2} \sin(t)\ dt$.

2) State and prove the Increasing Test.

3)

a) Sketch the graph of a function $f(x)$ with a unique root $x = 2$ such that Newton's method fails to find the root when we choose the initial approximation $x_1 = 4$.

b) Use Newton's method to find a root of the equation $\sin x = x^2 - 2$ correct to 2 decimal places.

4) State the definition of a differentiable function. Prove that if f is differentiable at a then f is continuous at a.

5) Consider the function

$$f(x) = \begin{cases} x+2, & \text{if } -2 \le x \le -1, \\ 1, & \text{if } -1 \le x \le 1; \\ \frac{-1}{2}x + \frac{3}{2}, & \text{if } 1 \le x \le 3. \end{cases}$$

a) Sketch the graph of $y = f(x)$.

b) Find $\int_{-2}^{0} f(x)\,dx, \int_{0}^{3} f(x)\,dx.$

6) Consider the following table of velocity/time data over a period of one minute. Assume that the velocity function is increasing over this time.

time in seconds	0	10	20	30	40	50	60
velocity in meters per second	0	1	3	4	6	9	12

a) Sketch a graph of $y = v(t)$.

b) Using left endpoints of ten-second intervals, estimate the distance travelled during this minute. State whether your estimate is too high or too low for the actual distance travelled, and explain your reasoning.

c) Using the given data, calculate a better estimate of the distance travelled. Justify that your method is an improvement.

7) Let $f(x) = 2^{|x^3+x|}$.

a) Calculate $f(-1), f(0)$. Deduce that $f'(x) = -3$ for some x in $(-1, 0)$. Explain your reasoning.

b) Calculate $f(1)$. Can you deduce that $f'(x) = 0$ for some x in $(-1, 1)$? Explain your answer.