CSE702 Homework # 1 Winter 2015—2016

Justify each answer ! **Due: 2 Feb 2016**

IN THE FOLLOWING PROBLEMS, IF YOU ARE USING MATLAB, THEN BUIT-IN FUNCTIONS FOR THESE METHODS SHOULD NOT BE USED.

1. For a continuously differentiable function $f : \mathbb{R} \to \mathbb{R}$, the problem of evaluating a function $y = f(x)$ has a condition number, C. An approximate definition of this condition number is given by

$$C \approx \left| \frac{x f'(x)}{f(x)} \right|.$$

Show that for the inverse problem $x = f^{-1}(y)$, the condition number is

$$C \approx \left| \frac{f(x)}{x f'(x)} \right|.$$

2. What is the approximate condition number of the function $y = x^2$?

3. What is the approximate condition number of the function $y = \sin(x)$ at $x = 0, \pi/2, \pi$? Make some general comments on how the sensitivity of this function depends on x.

4. For a floating point system with $\beta = 2, p = 4, L = -2$ and $U = 2$, what are the overflow and underflow limits?

5. For a floating point system, if $\beta = 10$, what are the smallest values of p and U, and the largest value of L such that both 2365.27 and 0.0000512 can be represented *exactly* in a *normalized* floating-point system.

6. Suppose we are interested in finding the sole root ($x^* = 1.4$) of the equation

$$x^3 - 1.4x^2 + x - 1.4 = 0.$$

Write a code that implements the Newton-Raphson method with an initial guess of $x_0 = 1.85$ to solve this problem. Determine the number of iterations needed to obtain 12 digits of accuracy in the solution.

7. The Newton-Raphson method *usually* exhibits a quadratic convergence rate. This is noted by observing that the ratio of the absolute errors in two successive iterations observe the following pattern

$$\frac{e_{n+1}}{e_n^2} \approx M,$$

where $0 < M < 1$. For the problem given in question 6, prove the quadratic error behaviour and determine the value of M.

8. The Newton-Raphson and Secant methods both use a straight line approximation to a function to determine the new guess at each successive iteration. However, one could equally use a quadratic (or any other) approximation to do this. Formulate a iteration scheme that uses a quadratic approximation to the function, and three function evaluations, to update the guess at each iteration — give a brief explanation of how you formulated the iteration scheme. Implement this scheme to find the root of the function $f(x) = x\sin(x) - 1$ in the interval $x \in [0, 2]$ with the stopping condition $|f(x_k)| < 10^{-10}$. Determine the number of iterations needed to obtain convergence when the initial guess $x_0 = 0.5$. **Hint: you may wish to do some reading on Muller's method.**

9. In class, we wrote down (schematically) a code that implement's Doolittle's algorithm to obtain the LU factorisation of an $n \times n$, full rank matrix. However, we did not account for the possibility of encountering an $a_{kk} = 0$ during this process. Write a code that implements Doolittle's method, with row permutations such that division by the zero a_{kk} is avoided.

10. Use the program written in 9 to determine the LU factorisation of the following matrix:

$$\mathbf{A} = \begin{pmatrix} 4 & 12 & 8 & 4 \\ 1 & 3 & 18 & 9 \\ 2 & 9 & 20 & 20 \\ 3 & 11 & 15 & 14 \end{pmatrix},$$

11. Use Cholesky factorization to solve the system $\mathbf{A}\ \mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 30 & -20 & -10 \\ -20 & 55 & -10 \\ -10 & -10 & 50 \end{pmatrix},$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 80 \\ 0 \end{pmatrix}.$$

You may do this either by hand, or on the computer. If you do this by hand, submit your working, if you use the computer, submit the code you wrote.