CSE702 **Homework # 2** **Winter 2015—2016**

Justify each answer ! **Due: 22 Feb 2016**

IN THE FOLLOWING PROBLEMS, IF YOU ARE USING MATLAB, THEN BUILT-IN FUNCTIONS FOR THESE METHODS SHOULD NOT BE USED.

1. Consider the following linear system of equations:

$$\begin{bmatrix} 2x_1 - x_2 \\ x1 + 4x_2 \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}.$$

Starting with an initial guess of the solution vector as $x^{(0)} = [0 \;\; 0]^T$,

(a) write a program that computes an approximate solution to this system using the Jacobi method.
(b) write a program that computes an approxiamte solution to this system using the Gauss-Siedel method.

In each case have the code compute 5 iterations, i.e., compute $x^{(1)}, ..., x^{(5)}$, and output the result of each iteration on the approximate solution vector to the screen.

2. Compute a singular value decomposition of the following two matricies:

$$\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

You may do this either by hand, or on the computer. If you do this by hand, submit your working, if you use the computer, submit your code.

3. Consider the following matrix:

$$\begin{bmatrix} 2 & 5 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

(a) Produce a plot of the characteristic polynomial of the matrix and use this plot to verify that $\lambda = -2, 1, 3$ are (at least) the approximate eigenvalues of the matrix.
(b) Write a code that implements the Rayleigh-Quotient iteration (shifted inverse power iteration with an RQ update on the eigenvalue at each step) to compute an approximation to the eigenvector associated with $\lambda = 1$. Begin the iterative process from the initial guess $\lambda^{(0)} = 0.8$. **Hint: you may wish to reuse the code you wrote to answer question 8 in the previous homework.**

4. Consider the following non-linear system of equations:

$$\begin{bmatrix} x_1 + 2x_2 - 2 \\ x_1^2 + 4x_2^2 - 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Starting with an initial guess of the solution vector as $x^{(0)} = [1 \;\; 2]^T$,

(a) write a program that computes an approximate solution to this system using Newton's method.
(b) write a program that computes an approxiamte solution to this system using Broyden's method.

Your code should output the approximate root as well as the iteration number at which the root is obtained. Write a sentence or two comparing the performance of the two methods.