$$\exp(x) = e^x \qquad \ln = \exp^{-1}$$

$$\boxed{f^{-1\prime}(f(x)) = \frac{1}{f'(x)}}$$

Examples:

$$\ln'(\exp(x)) = \frac{1}{\exp'(x)} = \frac{1}{\exp(x)}$$

i.e.

$$\ln'(y) = \frac{1}{y}.$$

$y = \exp(x)$ $\boxed{\ln'(x) = \frac{1}{x}}$

$$\arcsin'(\sin(x)) = \frac{1}{\cos(x)}$$

Now $\cos(x) = \sqrt{1 - \sin(x)^2}$, so

$$\boxed{\arcsin'(y) = \frac{1}{\sqrt{1-y^2}}}$$

$$\arctan'(\tan(x)) = \cos^2(x) = \frac{1}{1+\tan^2(x)}$$

$$\tan' = \frac{1}{\cos^2}$$

$$\arctan'(y) = \frac{1}{1+y^2}$$

$$\frac{d}{dx}\left(\left(e^{\ln x}\right)^t\right)$$

Power rule: For t a real number,

$$\frac{d}{dx}x^t = \cancel{\frac{d}{dx}\left(e^{\ln x}\right)^t} = \frac{d}{dx}e^{t\ln x} = \frac{t}{x}e^{t\ln x} = tx^{t-1}$$

chain rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

Exponentials again: We saw $\frac{d}{dx}e^x = e^x$.
Now $b^x = e^{\ln(b^x)} = e^{x\ln b}$, so

$$\frac{d}{dx}b^x = (\ln b)e^{x\ln b} = (\ln b)b^x$$

(Hence $\lim_{h\to 0}\frac{b^h-1}{h} = \ln b$)

$$\frac{d}{dx}x^{(t^2)} = t^2x^{t^2-1} \qquad \ln(a^b) = b\ln a$$

Midterm information: Newton 10th Oct

The test will be 90 minutes long

It will cover sections 1.6 (including inverse trig), 2.5, 2.7, 2.8, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.11, 4.1, 4.2, 4.8. There will be questions explicitly addressing Maple.

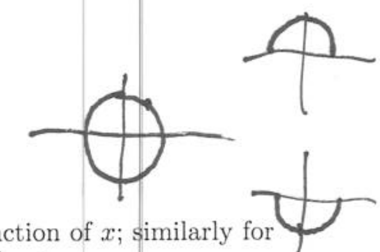
MVT

There will be two seatings for the exam on the Thursday evening; probably 6:30–8:00 and 8:15–9:45.

There will be an early seating for the exam on the Wednesday, probably at 2:30. Once the room and time assignments are posted, students who are unable to make the Thursday seatings will be able to request the early seating time.

Implicit differentiation

Suppose we know some relation between x and y, e.g.



$$x^2 + y^2 = 1.$$

Here, y isn't a function of x.

But if we restrict attention to $y \geq 0$, then y is a function of x; similarly for $y \leq 0$. These functions are *implicitly* defined by $x^2 + y^2 = 1$.

Restricting to a function in this way, it makes sense to differentiate with respect to x -

chain rule

$$0 = \frac{d}{dx}1 = \frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2 = 2x + \frac{dy}{dx}2y$$

and we conclude that, whichever function we chose,

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

for all x at which the function is differentiable.

Exercise: Confirm this agrees with the chain rule.

$$y = \sqrt{1-x^2}$$
$$y = -\sqrt{1-x^2}$$

Hyperbolic functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$