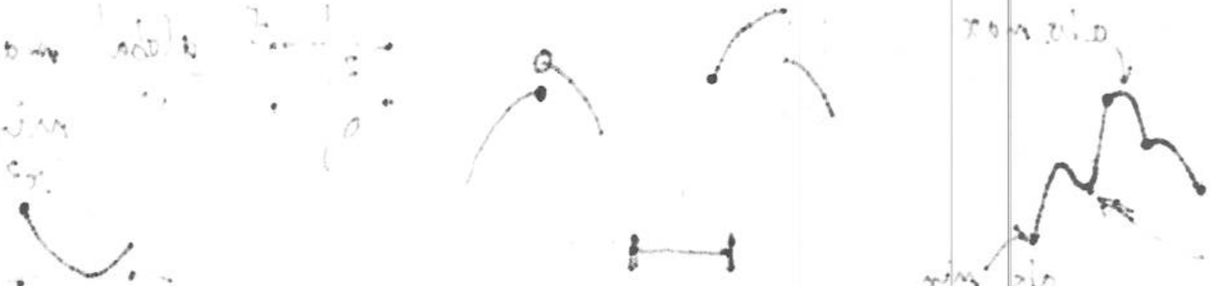
http://ms.mcmaster.ca/~mbays/teaching/lectureNotes/wk4/wk4.pdf



Minima and maxima

Definition: c is the <u>minimum value</u> of f if $f(x) = c$ for some x, and $f(x) \geq c$ for all x in $\mathrm{dom}(f)$. We say then that f has its <u>global</u> / <u>absolute minimum at</u> x, and that f <u>attains</u> its minimum value at x.

Examples: 0 is minimum value of x^2, but it has no maximum value.

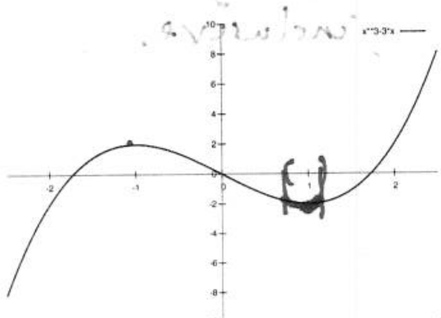
x^2 restricted to have domain $[-2, 2]$ has maximum value 4, attained at both -2 and 2.

e^x has no minimum value nor maximum value.

Definition: f has a <u>local minimum at</u> b in $\mathrm{dom}(f)$ if there is an interval around b on which f is defined and $f(x) \geq f(b)$.

(Here, an "interval around b" is one which contains b and which doesn't have b as an endpoint. e.g. $(-1, 1)$ is an interval around 0, but $[0, 1)$ isn't.)

Example: $x^3 - 3x$ has a local minimum at 1 and a local maximum at -1.



Local max and global max

$\cos(x)$ has a local maximum at $2k\pi$.

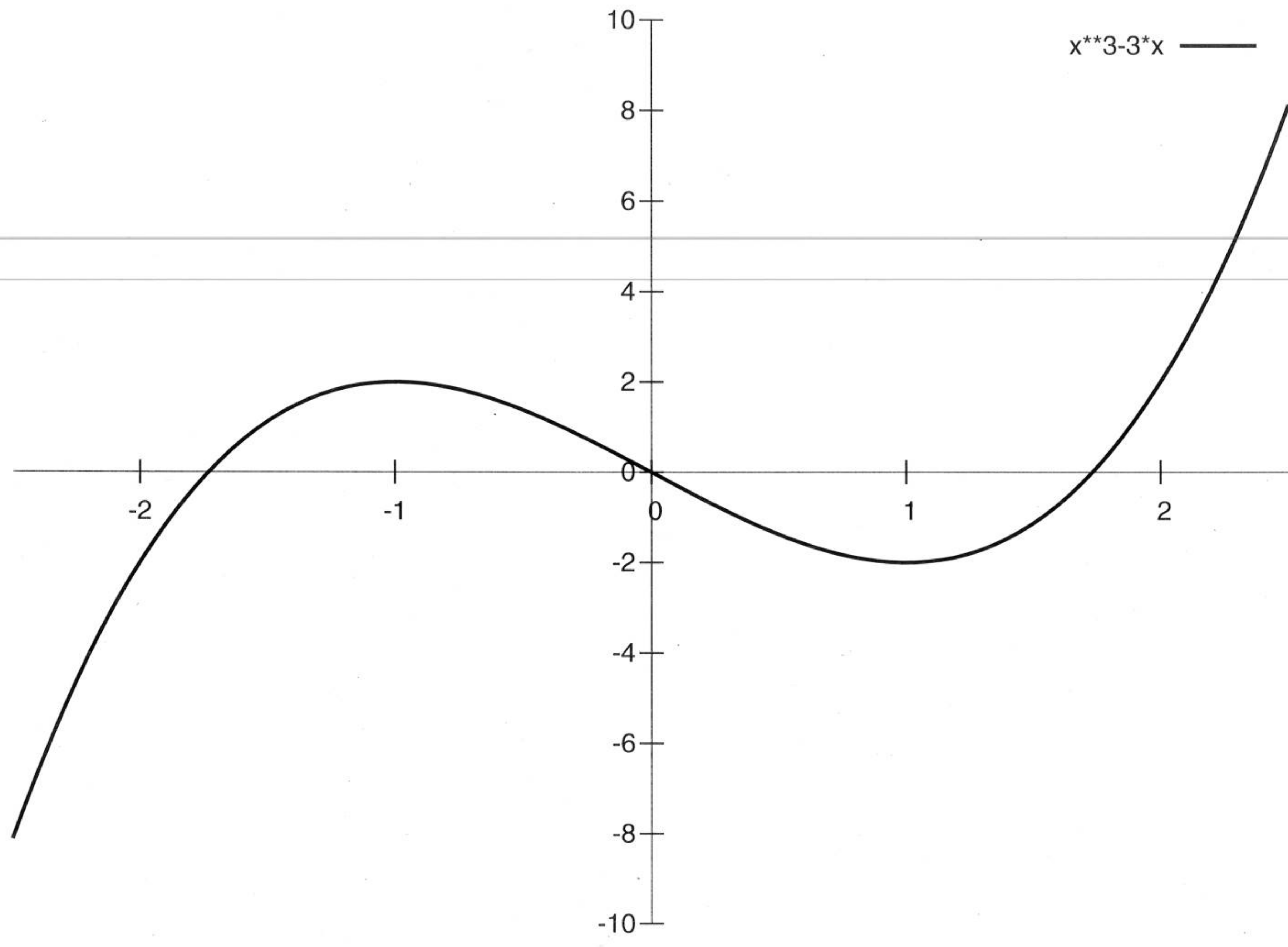
Definition: A <u>critical point</u> (or <u>critical number</u>) of f is a solution to $f'(x) = 0$.

Fact: If f has a local min/max at b and f is differentiable on an interval around b, then b is a critical point of f.

Remark: 0 is a critical point of x^3, but it is **not** a local min/max.

x^3

$\frac{d}{dx} x^3 = 3x^2$


x**3-3*x
10
8
6
4
2
0
-2
-4
-6
-8
-10
-2
-1
0
1
2