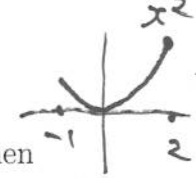
global max = 2
" min = 2



glob. max = 4

local min but not global min

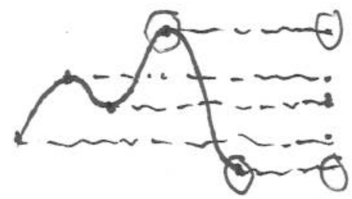
Fact: If dom(f) is a closed interval $[a, d]$ and f is continuous on $[a, d]$, then f has a minimum value and has a maximum value.

Remark: If f has a global minimum at b, then it has a local minimum at b unless b is in the boundary of dom(f).

e.g. d if $\mathrm{dom}(f) = [a, d]$

Conclusion: So if dom(f) is a closed interval $[a, d]$, and if f is continuous on $[a, d]$, then the minimum value of f is the least of the values at the endpoints and at any local minima.

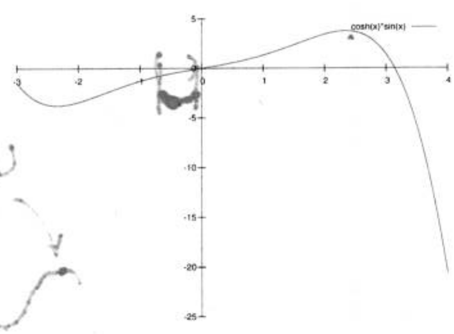
So if furthermore f is differentiable on (a, d), we can find the minimum value by



- Finding the zeros of f'
- Evaluating f at each such zero and at a and d
- The global minimum value is the least of these numbers.

Example: Find the minimum value and maximum value of $f(x) = \cosh(x)\sin(x)$ as x varies between -3 and 4, inclusive.

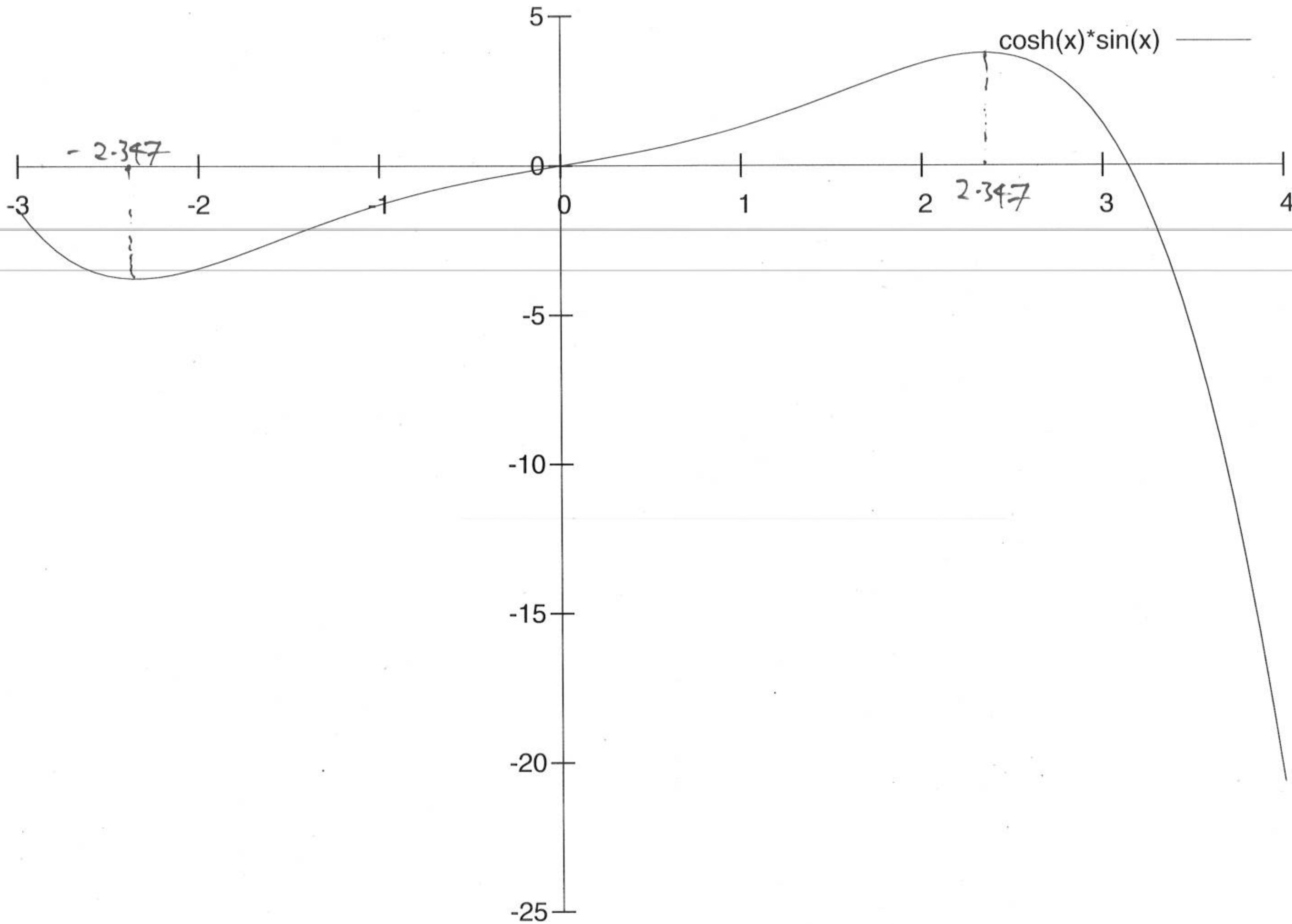
$[-3, 4]$



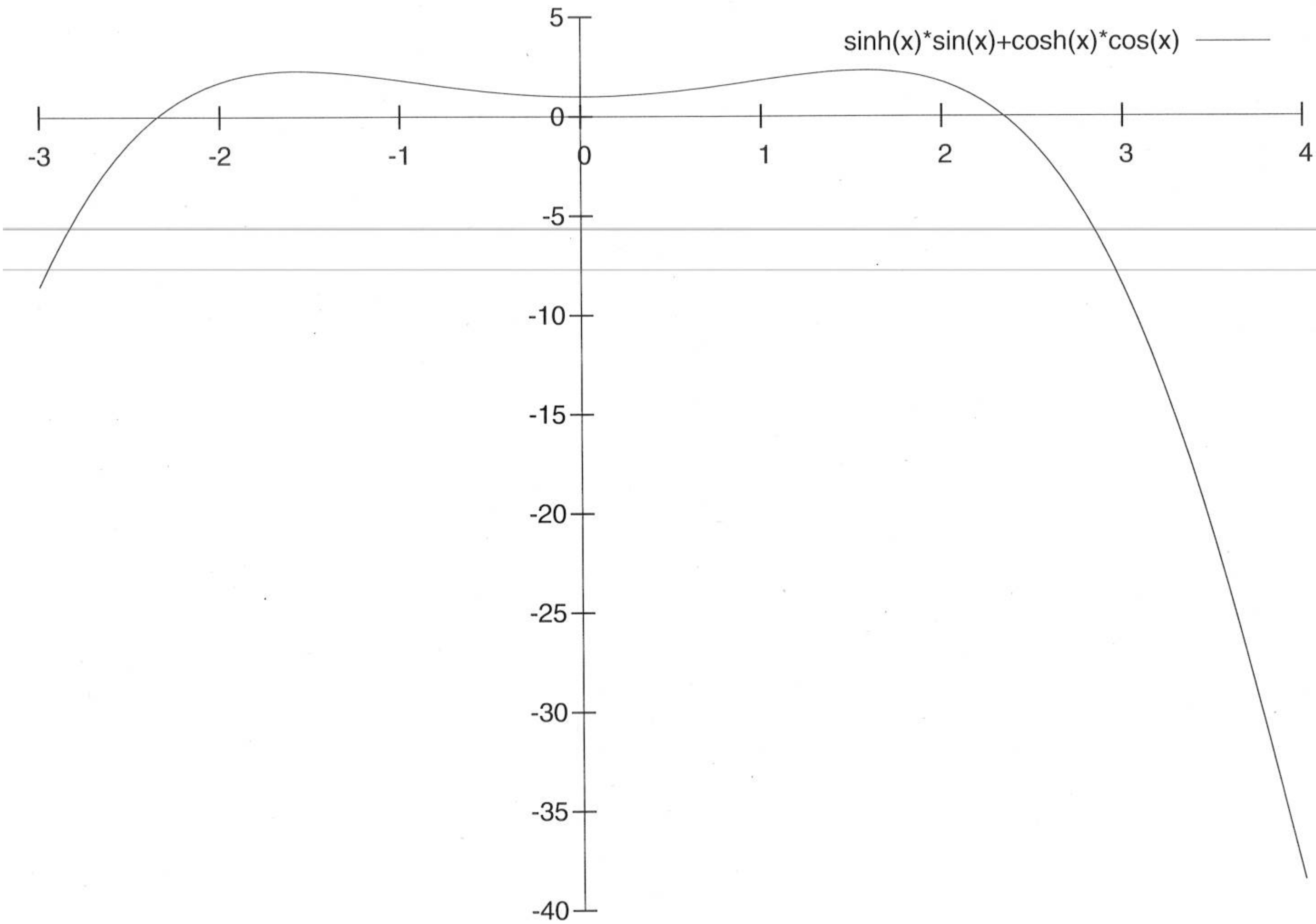
$f'(x) = \sinh(x)\sin(x) + \cosh(x)\cos(x)$

$f'(x) = \sinh(x)\sin(x) + \cosh(x)\cos(x)$. Use Newton's method to find zeroes.

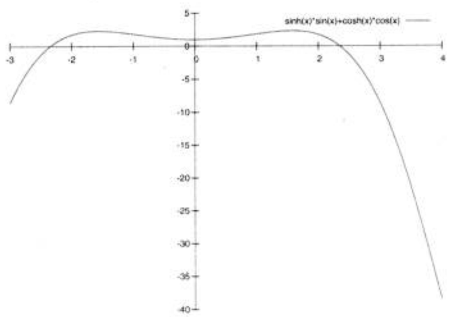
$$\frac{e^{x} - e^{-x}}{2} = \sinh(x)$$



cosh(x)*sin(x)
5
0
-5
-10
-15
-20
-25
-3
-2
-1
0
1
2
3
4
- 2.347
2.347



sinh(x)*sin(x)+cosh(x)*cos(x)
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-3
-2
-1
0
1
2
3
4



$(f''(x) = \cosh(x)\sin(x) + \sinh(x)\cos(x) + \sinh(x)\cos(x) - \cosh(x)\sin(x))$
Critical points are ± 2.347 critical points of f
$f(2.347) = 3.764$ (this is the global max)
$f(-2.347) = -3.764$
$f(-3) = -1.421$
$f(4) = -20.67$ (this is the global min)

MVT

Theorem [Rolle's Theorem]: "What goes up and then comes down must hover instantaneously in between"

Suppose f is differentiable on (a, d) and continuous at the endpoints a and d.

Suppose $f(a) = f(d)$.

Then f has a critical point in (a, d).

Proof: f has a maximum and a minimum value.

If they are equal, then f is constant, and any point is critical.

Else, f has a global min/max in (a, d), which is a local min/max, hence a critical point.

Theorem [MVT]: "Rolle's theorem at an angle"

Suppose f is differentiable on (a, d) and continuous at the endpoints a and d.

Let $s = \frac{f(d)-f(a)}{d-a}$

Midterm information:

The test will be 90 minutes long and consist of have 20 multiple choice questions.

It will cover sections 1.6 (including inverse trig), 2.5, 2.7, 2.8, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.11, 4.1, 4.8. There will be a question or two explicitly addressing Maple.

There will be two seatings for the exam on the Thursday evening; 6:30–8:00 and 8:15–9:45.

There will be an early seating for the exam on the Wednesday, at 2:30. If you are unable to make the Thursday seatings, you can request the early seating time on the homework website.

Midterm 1 syllabus in some detail

- Inverse functions - concept; invertible $\leftrightarrow$ 1-1.
- Limits - concept, calculation.
- Continuity.
- IVT - statement; use.
- Derivatives - limit definition; tangent lines; differentiability; diffble $\rightarrow$ conts; higher derivatives.
- Differentiation techniques: linearity; product rule; chain rule; differentiating polynomials, powers (x^b), exponentials (b^x), logarithms, $f(x)^{g(x)}$, trig/hyp, inverse trig/hyp. Newton's method
- trig/hyp: definitions (triangles for trig, $\cosh(x) = \frac{e^x+e^{-x}}{2}$ etc for hyp); fundamental identities ($\cos^2 + \sin^2 = 1 = \cosh^2 - \sinh^2$), inverses (arcfoo), differentiating.
- Minima and maxima: finding local and global mins/maxes; critical points