Mean Value Theorem

Theorem [Rolle's Theorem]: "What goes up and then comes down must hover instantaneously in between"

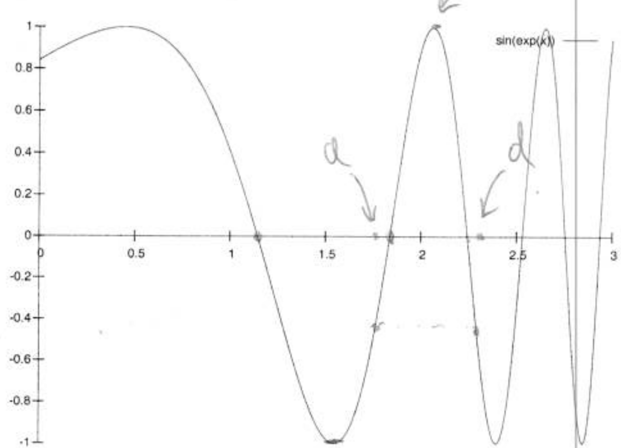
Suppose f is differentiable on (a, d) and continuous at the endpoints a and d. (hence continuous on $[a,d]$ since diff'ble $\Rightarrow$ cont's)

Suppose $f(a) = f(d)$.

Then f has a critical point in (a, d).

Example: $f(x) = \sin(e^x)$: $f(\ln(\pi)) = f(\ln(2\pi))$, so $f'(x) = 0$ for some x in $\cancel{(\pi, 2\pi)}$. $(\ln \pi, \ln 2\pi)$

critical point



critical point

Proof: f has a maximum and a minimum value.

If they are equal, then f is constant, and any point is critical.

Else, f has a global min/max in (a, d), which is a local min/max, hence a critical point.

Theorem [MVT]: "slanted Rolle"

Suppose f is differentiable on (a, d) and continuous at the endpoints a and d.

Let $s = \frac{f(d)-f(a)}{d-a}$.