Then for some b in (a, d),

$$f'(b) = s. = \frac{f(d) - f(a)}{d - a}$$

In other words: for some b in (a, d), the tangent line at b is parallel to the straight line between the points on the graph $(a, f(a))$ and $(d, f(d))$.

Example: $f(x) = \sin(e^x)$: $f\left(\ln\left(\frac{\pi}{2}\right)\right) = 1$, $f(\ln(\pi)) = 0$. So for some x in $\left(\ln\left(\frac{\pi}{2}\right), \ln(\pi)\right)$, $f'(x) = \frac{0-1}{\ln(\pi) - \ln\left(\frac{\pi}{2}\right)} = -1.44$.

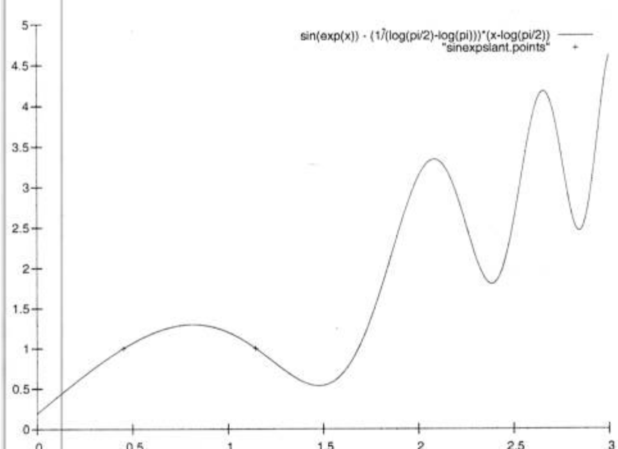
Proof: Let $g(x) = f(x) - s(x - a)$.

Then $g(a) = f(a)$ and $g(d) = f(d) - \frac{f(d)-f(a)}{d-a}(d - a) = f(d) - (f(d) - f(a)) = f(a) = g(a)$.

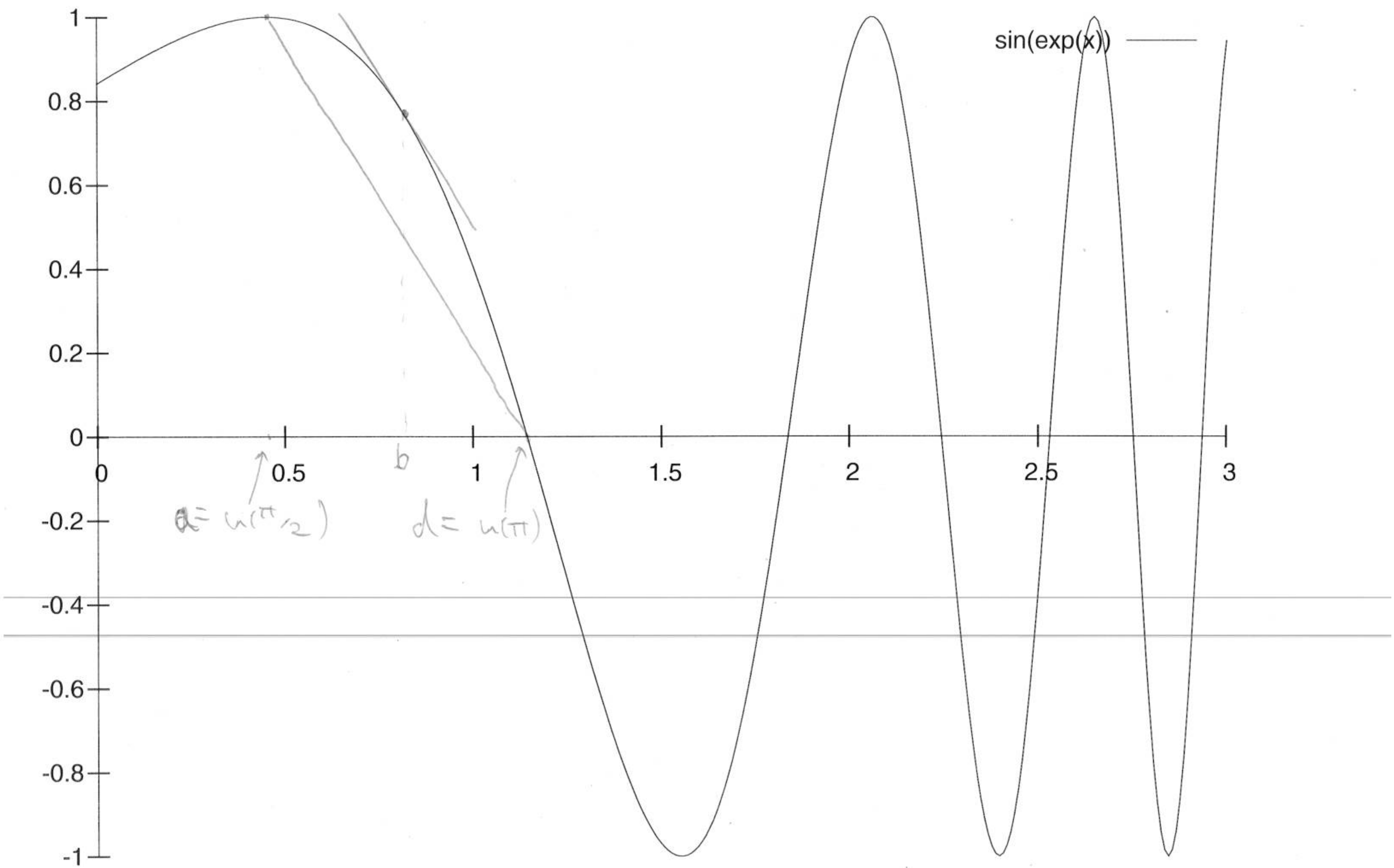
By Rolle's theorem, $g'(b) = 0$ for some b in (a, d).

But $g'(x) = f'(x) - s$, so

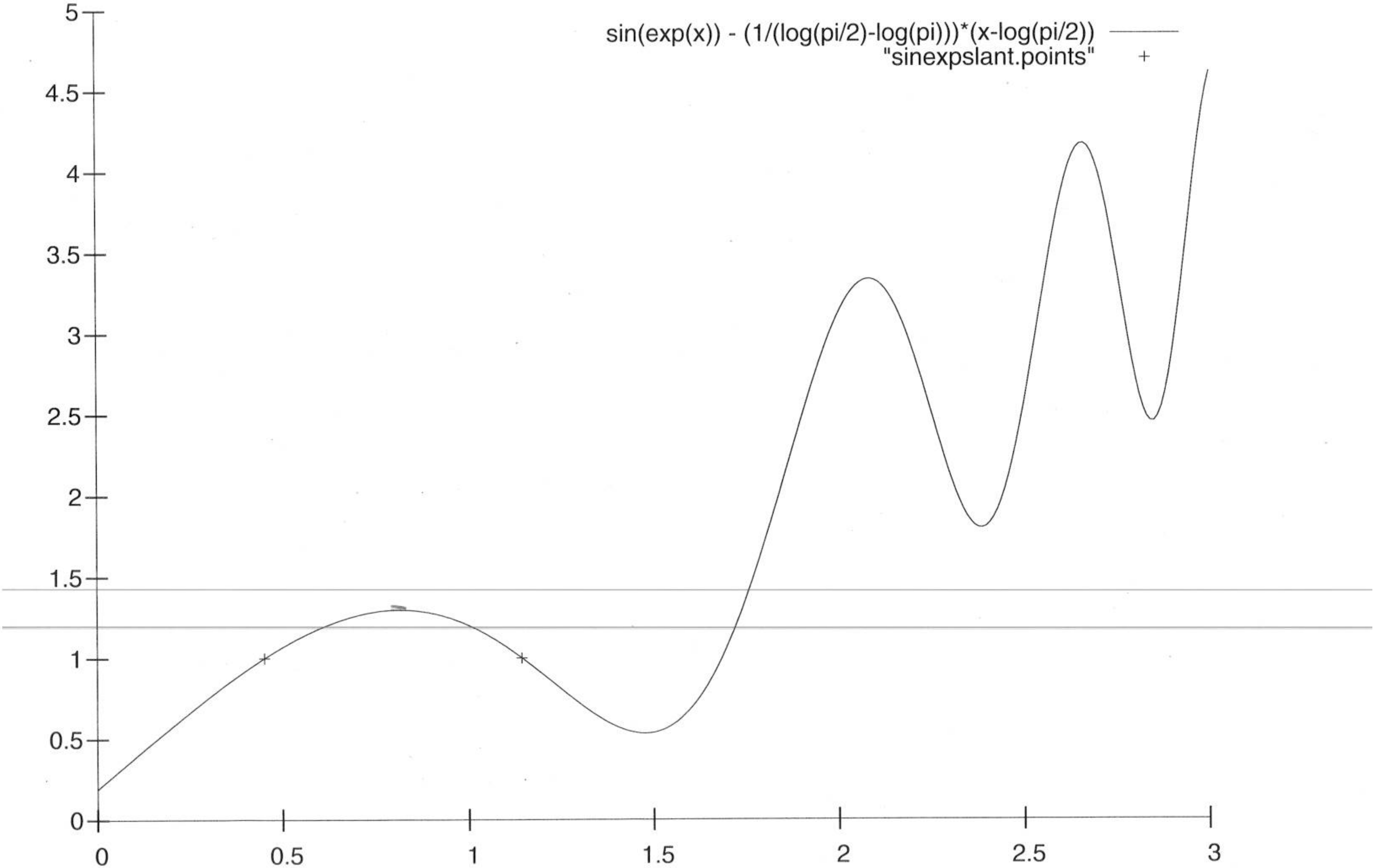
$$f'(b) = g'(b) + s = s.$$



We can use the MVT "backwards" to deduce information about $f(x)$ from information about $f'(x)$.



sin(exp(x))
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
b
a = u(π/2)
d = u(π)


sin(exp(x)) - (1/(log(pi/2)-log(pi)))*(x-log(pi/2))
"sinexpslant.points"
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3

Example: Suppose a train travels along a straight track, and $s(t)$ is its distance in metres from its starting station t seconds after it leaves.

If $s(600) = 10000$ and between $t = 600$ and $t = 1200$ the train's speed never drops below $30ms^{-1}$, what is the least $s(1200)$ could be?

Answer: $s(1200)$ can't be less than $s(600) + 30 * (1200 - 600) = 28000$, since otherwise by the MVT $s'(t)$ would, for some t between 600 and 1200, be less than $\frac{28000-10000}{1200-600} = 30$.

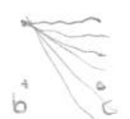
~~A≠V~~ $a = 600$ $d = 1200$

Increasing/Constant/Decreasing: Let $f(x)$ be a function. $s(a) = 10000$
suppose $s(d) < 28000$

(i) If $f'(x) > 0$ for all x in an interval, then $f(x)$ is increasing on the interval.

(ii) If $f'(x) = 0$ for all x in an interval, then $f(x)$ is constant on the interval.

(iii) If $f'(x) < 0$ for all x in an interval, then $f(x)$ is decreasing on the interval.



Proof:

(i) If b and c are in the interval with $b < c$ and $f(c) \leq f(b)$, then by MVT $f'(x) = \frac{f(c)-f(b)}{c-b} \leq 0$ for some x in $[b, c]$.

(ii) If b and c are in the interval with $b < c$ and $f(c) \neq f(b)$, then by MVT $f'(x) = \frac{f(c)-f(b)}{c-b} \neq 0$ for some x in $[b, c]$.

(iii) If b and c are in the interval with $b < c$ and $f(c) \geq f(b)$, then by MVT $f'(x) = \frac{f(c)-f(b)}{c-b} \geq 0$ for some x in $[b, c]$.

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Request for wednesday exam