





One-sided limits, $\lim_{x \rightarrow a^+}$ or $\lim_{x \rightarrow a^-}$, are also allowed.

Moreover, we have the same result if $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ are both $\pm\infty$ [even if one is $+\infty$ and the other $-\infty$], rather than both being 0.

[To see that it works for infinite limits: consider $\frac{1}{f(x)}$ and $\frac{1}{g(x)}$, which both tend to 0 at a]

Examples:

- $\lim_{x \rightarrow 0} \frac{\sin(x)}{x^2 - 4x} = \lim_{x \rightarrow 0} \frac{\cos(x)}{2x - 4} = -\frac{1}{4}$.
- $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{2x}{1} = 2$
- $\lim_{x \rightarrow +\infty} \frac{e^x}{x} = \lim_{x \rightarrow +\infty} \frac{e^x}{1} = +\infty$
- $\lim_{x \rightarrow +\infty} \frac{e^x}{x^2} = \lim_{x \rightarrow +\infty} \frac{e^x}{2x} = \lim_{x \rightarrow +\infty} \frac{e^x}{2} = +\infty$
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$$\begin{aligned} \lim_{x \rightarrow 0^+} x \ln x &= \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x}} \\ &= \lim_{x \rightarrow 0^+} \frac{\frac{d}{dx} \ln x}{\frac{d}{dx} \frac{1}{x}} \\ &= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} \\ &= \lim_{x \rightarrow 0^+} -x \\ &= 0 \end{aligned}$$

Sketching curves

Examples.