Example of graph we can't yet sketch:

$$f(x) = \frac{\sin(x)}{x^2 - 4x}$$

What happens near 0?

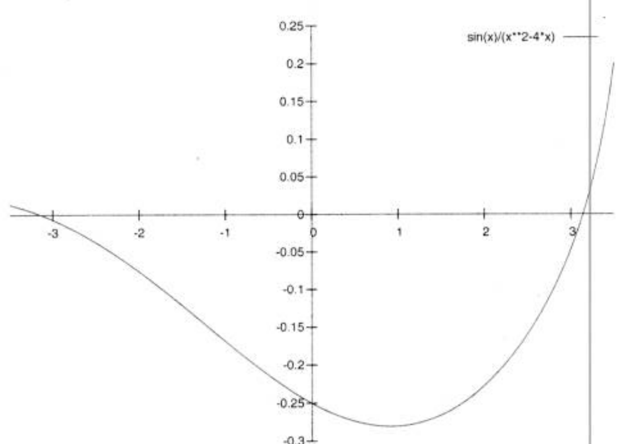
i.e., what is $\lim_{x\to 0} f(x)$?

L'Hôpital

Near 0, $\sin(x)$ is well approximated by x [this is the equation of the tangent line at 0, since $\sin'(0) = \cos(0) = 1$], and $x^2 - 4x$ is well approximated by $-4x$ [since $\frac{d}{dx}x^2 - 4x \upharpoonright_0 = 2x - 4 \upharpoonright_0 = -4$]

So near 0, $\frac{\sin(x)}{x^2-4x} \approx \frac{x}{-4x} = -\frac{1}{4}$.

So we expect $\lim_{x\to 0} \frac{\sin(x)}{x^2-4x} = -\frac{1}{4}$**. Indeed:**



This kind of reasoning yields: Theorem: [L'Hôpital's Rule] Suppose $\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$.

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$, assuming the right hand limit exists.

Here, a is allowed to be $+\infty$ or $-\infty$, and so is the right hand limit.

$\lim_{x\to 0} \frac{x+2}{x+1} = 2$

$\neq$

$\lim_{x\to 0} \frac{1}{1} = 1$