-

$$\begin{aligned}
\lim_{x\to 0^+} (1+x)^{\frac{1}{x}} &= \lim_{x\to 0^+} \left(e^{\ln(1+x)}\right)^{\frac{1}{x}} \\
&= \lim_{x\to 0^+} e^{\frac{1}{x}\ln(1+x)} \\
&= e^{\left(\lim_{x\to 0^+} \frac{\ln(1+x)}{x}\right)} \\
&= e^{\left(\lim_{x\to 0^+} \frac{\frac{1}{1+x}}{1}\right)} \\
&= e^1 = e
\end{aligned}$$

$\lim_{x\to 0^+} \ln(1+x) = \ln 1 = 0$

$\lim_{x\to 0^+} x = 0$

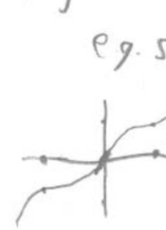
$$1.00001^{100000} = 2.7182682371922975$$

$e = 2.71828$

Sketching curves

Things to work out (or estimate) when sketching a graph:

- zeroes;
- symmetry; e.g. even $f(x)=f(-x)$ or odd $f(x)=-f(-x)$
  e.g. cos, x^2, x^4 e.g. sin, x^3, x^5
- inc/dec; min/max; f'
- concavity; f''



- behaviour at $\pm\infty$;
- limits at points of undefinedness / discontinuity.

Examples:

- $$f(x)=\frac{x^2-1}{(x-1)(x-2)(x-3)}$$
  $$=\frac{(x+1)(x-1)}{(x-1)(x-2)(x-3)}$$
  $$=\frac{x+1}{(x-2)(x-3)}$$

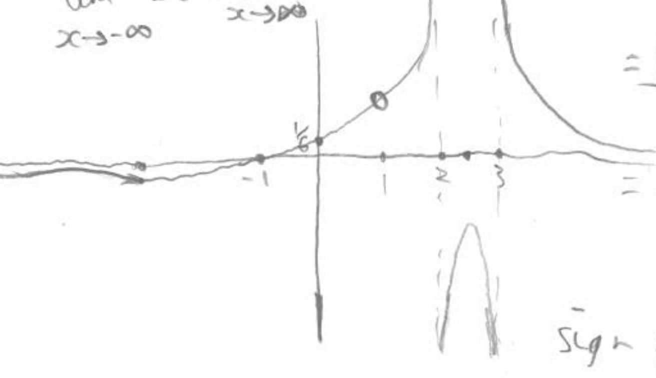
Hole at $x=1$

$\lim_{x\to 2^-} f(x)=+\infty=\lim_{x\to 3^+} f(x)$

$\lim_{x\to 2^+} f(x)=-\infty=\lim_{x\to 3^-} f(x)$

zeroes: $f(x)=0 \iff x=-1$

$\lim_{x\to-\infty}=0=\lim_{x\to\infty}$



$$f'(x)=\frac{(x-2)(x-3)-(x+1)(2x-5)}{(x-2)^2(x-3)^2}$$
$$=\frac{(x^2-5x+6)-(2x^2-3x-5)}{(x-2)^2(x-3)^2}$$
$$=\frac{-(x^2+2x-11)}{(x-2)^2(x-3)^2}$$

$x^2+2x-11=0$

$$\iff x=\frac{-2\pm\sqrt{4+44}}{2}$$
$$=\frac{-2\pm\sqrt{48}}{2}$$

approx $\frac{-9}{2}$, $\frac{5}{2}$

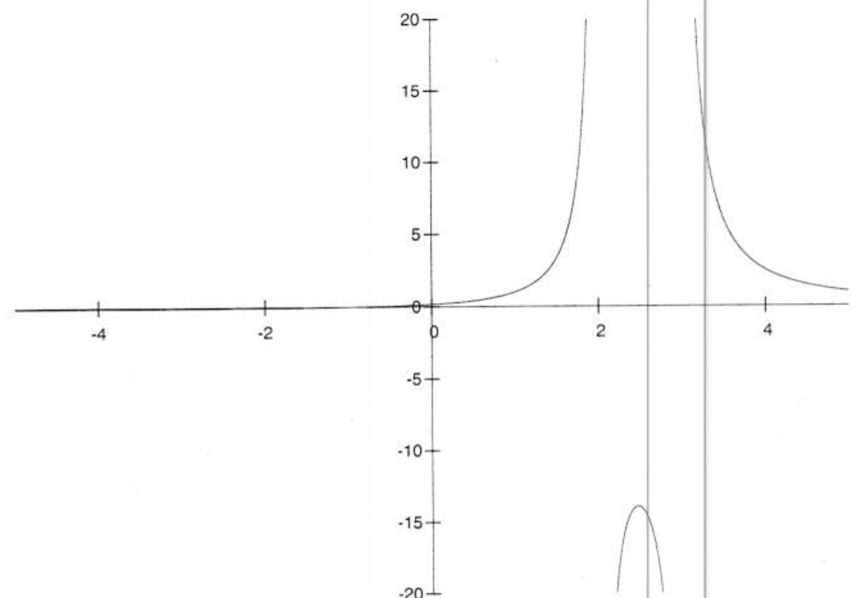
sign of $f'(x)$

= sign of $-(x^2+2x-11)$

because bottom is always +ve

So f is decreasing on $(-\infty, \sim\frac{-9}{2})$ and on $(\sim\frac{5}{2}, +\infty)$

increasing on $(-\frac{9}{2}, \frac{5}{2})$



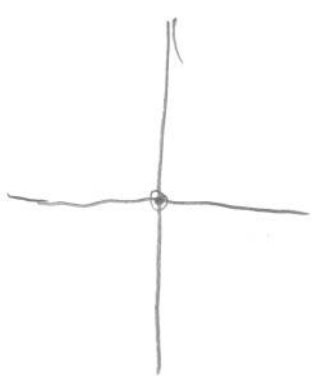
- $f(x) = e^{\frac{1}{x}}$

$\lim_{x \to 0^-} f(x) = \lim_{y \to -\infty} e^y = 0$

$f(x) = 0$ never!

$\lim_{x \to 0^+} f(x) = +\infty$

$f'(x) = \frac{-e^{1/x}}{x^2}$



never 0
top is always negative
bottom is always +ve
so $f'(x)$ is always -ve
so $e^{1/x}$ is decreasing everywhere