4.19 What’s wrong? In each of the following scenarios, there is something wrong. Describe what is wrong and give a reason for your answer.

a) If two events are disjoint, we can multiply their probabilities to determine the probability that they will both occur.

b) If the probability of A is 0.6 and the probability of B is 0.5, the probability of both A and B happening is 1.1.

c) If the probability of A is 0.35, then the probability of the complement of A is −0.35.

**Solution**

a) Two disjoint events can never occur together. The probability that one or the other occurs is the sum of their individual probabilities. (Multiplication is appropriate for independent events.)

b) Probabilities must be no more than 1; \( P(A \text{ and } B) \) will be no more than 0.5.

c) \( P(A^c) = 1 - 0.35 = 0.65 \) & can't be negative

4.25 Distribution of blood types. All human blood can be “ABO-typed” as one of O, A, B, or AB, but the distribution of the types varies a bit among groups of people. Here is the distribution of blood types for a randomly chosen person in the United States:

<table>
<thead>
<tr>
<th>Blood type</th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. probability</td>
<td>0.42</td>
<td>0.11</td>
<td>?</td>
<td>0.44</td>
</tr>
</tbody>
</table>

a) What is the probability of type AB blood in the United States?

b) Maria has type B blood. She can safely receive blood transfusions from people with blood types O and B. What is the probability that a randomly chosen person from the United States can donate blood to Maria?

**Solution:**

(a) \( P(A) + P(B) + P(AB) + P(O) = 1 \)

\[ \Rightarrow 0.42 + 0.11 + x + 0.44 = 1 \]

\[ \Rightarrow x = .03 \]

(b) Here we can use the addition rule for disjoint events to calculate \( P(O \text{ or } B) \).

\[ P(O \text{ or } B) = P(O) + P(B) = .44 + .11 = .55 \]
4.26 Blood types in Ireland. The distribution of blood types in Ireland differs from the U.S. distribution given in the previous exercise:

<table>
<thead>
<tr>
<th>Blood type</th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ireland probability</td>
<td>0.35</td>
<td>0.10</td>
<td>0.03</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Choose a person from the United States and a person from Ireland at random, independently of each other.

a) What is the probability that both have type O blood?
b) What is the probability that both have the same blood type?

Solution
(a) Let \( E \) be the event that the person we choose in the US has type O blood, and let \( F \) be the event that the person we choose in Ireland has type O blood. Here we can use the multiplication rule for independent events to compute \( P( E \text{ and } F) \).

\[
P( E \text{ and } F) = (.44) \times (.52) = .2288
\]

(b) \( P(\text{"same blood type"}) = P(AA) + P(BB) + P(AB \text{ AB}) + P(OO). \)

We already know \( P(OO) \) from part (a). Work as in part (a) to find \( P(AA), P(BB), P(AB \text{ AB}) \).

\[
P(\text{"same blood type"}) = 0.147 + 0.011 + 0.0009 + 0.2288 = 0.3877
\]

4.28 French and English in Canada. Canada has two official languages, English and French. Choose a Canadian at random and ask, “What is your mother tongue?” Here is the distribution of responses, combining many separate languages from the broad Asian/Pacific region:

<table>
<thead>
<tr>
<th>Language</th>
<th>English</th>
<th>French</th>
<th>Asian/Pacific</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.59</td>
<td>?</td>
<td>0.07</td>
<td>0.11</td>
</tr>
</tbody>
</table>

(a) What probability should replace “?” in the distribution?
(b) What is the probability that a Canadian’s mother tongue is not English? Explain how you computed your answer.

Solution
(a) The given probabilities have sum 0.77, so \( P(\text{French}) = 0.23. \)

(b) \( P(\text{not English}) = 1 - 0.59 = 0.41, \) using Rule 4. (Or, add the other three probabilities.)
4.29 Education levels of young adults. Choose a young adult (age 25 to 34 years) at random. The probability is 0.12 that the person chosen did not complete high school, 0.31 that the person has a high school diploma but no further education, and 0.29 that the person has at least a bachelor’s degree.

(a) What must be the probability that a randomly chosen young adult has some education beyond high school but does not have a bachelor’s degree?
(b) What is the probability that a randomly chosen young adult has at least a high school education?

Solution

<table>
<thead>
<tr>
<th>Ed Level</th>
<th>No HS Diploma</th>
<th>HS only</th>
<th>HS+</th>
<th>BA+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.12</td>
<td>0.31</td>
<td>?</td>
<td>0.29</td>
</tr>
</tbody>
</table>

(a) The given probabilities have sum 0.72, so this probability must be 0.28.
(b) \( P(\text{at least a high school education}) = 1 - P(\text{has not finished HS}) = 1 - 0.12 = 0.88 \). (Or add the other three probabilities.)

4.32 Roulette. A roulette wheel has 38 slots, numbered 0, 00, and 1 to 36. The slots 0 and 00 are colored green, 18 of the others are red, and 18 are black. The dealer spins the wheel and at the same time rolls a small ball along the wheel in the opposite direction. The wheel is carefully balanced so that the ball is equally likely to land in any slot when the wheel slows. Gamblers can bet on various combinations of numbers and colors.

(a) What is the probability that the ball will land in any one slot?
(b) If you bet on “red,” you win if the ball lands in a red slot. What is the probability of winning?
(c) The slot numbers are laid out on a board on which gamblers place their bets. One column of numbers on the board contains all multiples of 3, that is, 3, 6, 9,…, 36. You place a “column bet” that wins if any of these numbers comes up. What is your probability of winning?

Solution

(a) \( P(0) = P(00) = P(1) = P(2) = \ldots = P(36) = 1/38 = 0.0263 \)
(b) \( P(\text{Red}) = 18/38 = .4737 \)
(c) \( P(\text{multiple of 3}) = P(3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36) = \frac{12}{38} = 0.3158 \)

4.34 PINs. The personal identification numbers (PINs) for automatic teller machines usually consist of four digits. You notice that most of your PINs have at least one 0, and you wonder if the issuers use lots of 0s to make the numbers easy to remember. Suppose that PINs are assigned at random, so that all four-digit numbers are equally likely.

(a) How many possible PINs are there?
(b) What is the probability that a PIN assigned at random has at least one 0?

Solution

(a) \( 10 \times 10 \times 10 \times 10 = 10000 \)
(b) \( P(\text{at least one zero}) = 1 - P(\text{no zero}) = 1 - \left(\frac{9 \times 9 \times 9 \times 9}{10000}\right) = 0.3439 \)

4.39 Random walks and stock prices. The “random walk” theory of securities prices holds that price movements in disjoint time periods are independent of each other. Suppose that we record only whether the price is up or down each year, and that the probability that our portfolio rises in price in any one year is 0.65. (This probability is approximately correct for a portfolio containing equal dollar amounts of all common stocks listed on the New York Stock Exchange.)

(a) What is the probability that our portfolio goes up for three consecutive years?
(b) If you know that the portfolio has risen in price two years in a row, what probability do you assign to the event that it will go down next year?
(c) What is the probability that the portfolio’s value moves in the same direction in both of the next two years?

Solution

The table shows the probabilities that the portfolio will rise or fall in any given year.

<table>
<thead>
<tr>
<th>Portfolio of stocks</th>
<th>Portfolio price rises</th>
<th>Portfolio price declines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.65</td>
<td>0.35</td>
</tr>
</tbody>
</table>

(a) \( P(\text{portfolio rises for 3 consecutive years}) = 0.65 \times 0.65 \times 0.65 = 0.2746 \)
(b) 0.35 - No reason that the probability of the portfolio falling in price should change because the price of the portfolio has gone up the previous two years.
(c) \( P(\text{portfolio moves in same direction in next two years}) \)

\[ = P(\text{Portfolio price rises on both years OR portfolio price falls on both years}) \]

\[ = P(\text{Portfolio price rises on both years}) + P(\text{portfolio price falls on both years}) \]
\[(0.65)^2 + (0.35)^2 = 0.545\]